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Correlation between hazardous rate and variables in an inventory model of two-parameter exponential distribution with finite production rate

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Abstract
In the present paper, correlation coefficients are calculated between hazardous rate and various variables in an inventory model of two-parameter negative exponential distribution. The production and demand rates are constant. Shortages are not allowed. It is observed that the time of production, the time of one cycle and the maximum inventory level have negative high correlation with the hazardous rate and the average total cost per unit time has positive high correlation with the hazardous rate.

Keywords: correlation, finite production rate, hazardous items, inventory model, two-parameter exponential distribution

1. Introduction

Inventory is defined as the stock of items to satisfy the future demands. Harris [1] developed the mathematical model to decide the number of products at ones. He also gave the concept of economic order quantity (EOQ). After him many mathematical models have been developed for controlling the inventory. In several exciting models, it is assumed that the products have infinite shelf time. But actually deterioration plays a vital role in inventory. Deterioration is defined as decay, spoilage, loss of utility of products etc. The process of deterioration is observed in volatile liquids, beverages, medicines, blood components, sweets, fruits and vegetables. There are many other products in the real world which deteriorate with a significant rate. So it should not be neglected in the decision process of production lot size.

In recent years, mathematical ideas have been used in different areas in real life problems, particularly for controlling inventory. When the items of the commodity are kept in stock as an inventory for fulfilling the future demand, there may be deterioration of items in the inventory system.

At the end of the storage period, deterioration is studied by Whit in [2] for the fashion goods. Ghare and Schrader [3] analyzed the problem of decaying inventories exponentially and developed an EOQ model with constant demand. Covert and Philip [4] extended Ghare and Schrader's model by considering a two parameter Weibull's distribution for variable rate of deterioration. Shah and Jaiswal [5] developed an order-level inventory model for a system with constant rate of deterioration. Aggarwal [6] modified Shah and Jaiswal's model in calculating the average inventory holding cost. Yang and Wee [7] developed an integrated multi-lot-size production inventory model for deteriorating item. Sharma *et al*. [8] gave a deterministic production inventory model for deteriorating products with exponentially declining demand and shortages. Baten and Kamil [9] studied the inventory management systems with two-parameter exponential distributed hazardous items in which production and demand rates are constant. Manna and Chiang [10] presented economic production quantity models for deteriorating items with ramp type demand. Sharma *et al*. [11] developed an inventory model for hazardous items of two-parameter exponential distribution with finite production rate. Sharma and Muhammad [12] analysed correlation between parameters and variables of EOQ models without shortages for hazardous items. Sharma and Muhammad [13] studied an EOQ model for hazardous items with uniform rate of demand. Sharma and Muhammad [14] developed an EOQ model with instantaneous production for hazardous items of two-parameter exponential distribution. Sharma and Muhammad [15] analyzed sensitivity of inventory model for deteriorating items with on-hand inventory dependent demand rate and infinite production rate without shortages.

Some commodities were observed to shrink with time by a proportion which can be approximated by a negative exponential function of time. The probability density function of a two-parameter exponential distribution is given by

$$f(t;\mu,\eta) = \frac{1}{\eta} e^{-\frac{(t-\mu)}{\eta}}, \ t \geq \mu, \eta > 1,$$

Where μ is the location parameter and η is the scale parameter.
The unreliability function is given by

$$F(t;\mu,\eta) = 1 - e^{-\frac{(t-\mu)}{\eta}}.$$

The failure or hazard rate function of on-hand inventory is given by

$$H(t;\mu,\eta) = \frac{f(t;\mu,\eta)}{1-F(t;\mu,\eta)} = \frac{1}{\eta}, t \geq \mu, \eta > 1.$$

So the hazardous rate followed by the two-parameter exponential distribution is constant.
In this paper, the objective is to correlate hazardous rate and various variables in an inventory model of two-parameter negative exponential distribution. The conclusion is illustrated by numerical examples and graphs in various situations.
The Karl-Pearson's coefficient of correlation between two variables x and y is given by

$$r_{xy} = \frac{covariance(x,y)}{\sqrt{\text{variance}(x)}\sqrt{\text{variance}(y)}}, -1 \leq r_{xy} \leq 1.$$

2. Assumptions and Notations

1. d is the rate of demand, which is known.
2. I(t) is the on hand inventory at any time t.
3. R is the finite production rate per unit time.
4. $H(t;\mu,\eta) = \frac{1}{\eta}$ Is constant hazardous rate per unit time? It follows two-parameter exponential distribution.
5. C_p is the production cost of one item.
6. C_h is the inventory holding cost coefficient per unit time.
7. C_o is the operating cost per order.
8. C is the average total cost per unit time.
9. T is the time of one cycle.
10. t_1 is the time of production.
11. I_1 is the maximum inventory level
12. Lead time is zero.
13. Shortages are not allowed.
14. The inventory system deals with only one item.
15. d, R, p, C_h, C_o, C, I, t_1, $I_1 > 0$; I(t), $t \geq 0$; $R > d$.

3. Mathematical Model

The differential equations describing the behavior of the system are given by

$$\frac{dI(t)}{dt} = R - d - \frac{1}{\eta} I(t) \text{ for } 0 \leq t \leq t_1 \qquad \dots (3.1)$$

and

$$\frac{dI(t)}{dt} = -d - \frac{1}{\eta} I(t) \text{ for } t_1 \leq t \leq T \qquad \dots (3.2)$$

with boundary conditions I(0) = 0, $I(t_1) = I_1$ and I(T) = 0 … (3.3)
The solution of above system is

$$I(t) = \begin{cases} \eta(R-d)\left(1-e^{-\frac{t}{\eta}}\right), 0 \leq t \leq t_1 \\ \eta d\left(e^{\frac{(T-t)}{\eta}}-1\right), \ t_1 \leq t \leq T \end{cases} \qquad \dots (3.4)$$

$$\text{with } I_1 = \eta(R-d)\left(1-e^{-\frac{t_1}{\eta}}\right) \qquad \dots (3.5)$$

$$\text{and } T = \eta \cdot \log\left[\frac{R}{d}\left(e^{\frac{t_1}{\eta}}-1\right)+1\right] \qquad \dots (3.6)$$

The situation of inventory is illustrated in figure (1).

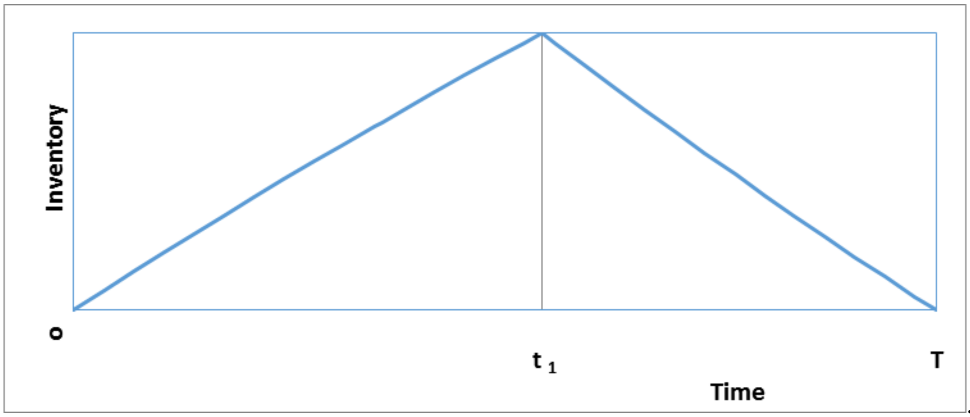

Fig 1

Neglecting the terms containing η with degree greater than or equal to 2, we have

$$I(t) = \begin{cases} (R-d)\left(t - \frac{t^2}{2\eta}\right), & 0 \le t \le t_1 \\ d\left((T-t) + \frac{(T-t)^2}{2\eta}\right), & t_1 \le t \le T. \end{cases} \quad \ldots (3.7)$$

The inventory holding cost for one cycle is

$$HC = C_h \int_0^T I(t)dt$$

Using equation (3.7), we get

$$HC = \frac{C_h}{6\eta}[(R-d)(3\eta t_1^2 - t_1^3) + d\{3\eta(T-t_1)^2 + (T-t_1)^3\}]$$

The number of units produced in time $t_1 = Rt_1$
The production cost per production run is

$$PC = C_p R t_1$$

The set-up cost per cycle is

$$SC = C_o$$

The total cost for one cycle of time T is

$$TC = HC + PC + SC$$

Hence the average total cost per unit time is

$$C = \frac{TC}{T}$$

$$= \frac{C_h}{6\eta T}[(R-d)(3\eta t_1^2 - t_1^3) + d\{3\eta(T-t_1)^2 + (T-t_1)^3\}] + \frac{C_p R t_1}{T} + \frac{C_o}{T} \quad \ldots (3.8)$$

By equations (3.6) and (3.8), it is clear that C is a function of only one variable t_1. For C to be minimum, $\frac{dC}{dt_1} = 0$ and $\frac{d^2C}{{dt_1}^2} > 0$. By $\frac{dC}{dt_1} = 0$, we have

$$3\eta\left[C_h d\{(R-d)(2\eta-t_1)t_1 + 2\eta C_p R\}\left\{\frac{R}{d}\left(e^{\frac{t_1}{\eta}}-1\right)+1\right\} + C_h d(R-d)(2\eta+T-t_1)(T-t_1)\right]\log\left\{\frac{R}{d}\left(e^{\frac{t_1}{\eta}}-1\right)+1\right\} - R\left[C_h\{(R-d)(3\eta-t_1){t_1}^2 + d(3\eta+T-t_1)(T-t_1)^2\} + 6\eta\left(C_p R t_1 + C_o\right)\right]e^{\frac{t_1}{\eta}} = 0 \qquad (3.9)$$

Solving equation (3.9) by Newton-Raphson method, the value of t_1 is obtained numerically up to desired accuracy. That value of t_1, by which $\frac{d^2C}{{dt_1}^2} > 0$, gives the minimum value of C. Hence the optimum values of the cycle time T, inventory level I_1 and minimum average cost are obtained.

4. Numerical Examples

1. Let us consider the values of parameters d = 6 units/week, R = 16 units/week, C_p = \$ 16/unit, C_o = \$240/set up, C_h = \$ 4 per unit per week, $H = \frac{1}{\eta} = 0.06$. Then the following results were obtained from our inventory model: t_1 = 2.05 weeks, T = 4.99 weeks, I_1 = 19.3 units and minimum C = \$ 191.43. The inventory level I(t) at any time t is shown in figure (2).

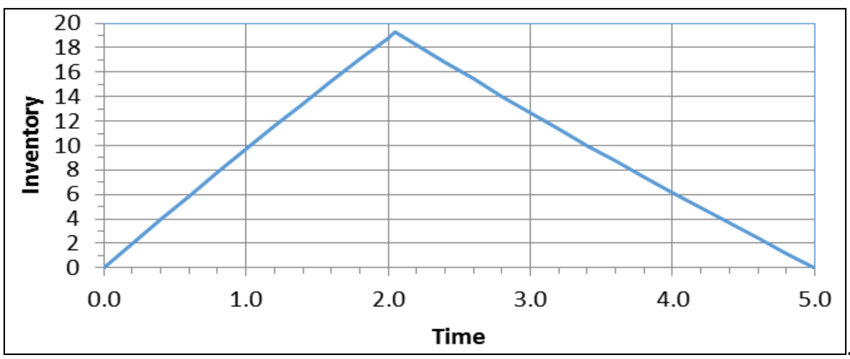

Fig 2

2. For d = 8 units/week and other values same as example 1, the results are t_1 = 2.66 weeks, T = 4.96 weeks, I_1 = 19.68 units and minimum C = \$ 225.33.
3. For R = 20 units/week and other values same as example 1, the results are t_1 = 1.54 weeks, T = 4.65 weeks, I_1 = 20.57 units and minimum C = \$ 197.67.
4. For C_p = \$ 12/unit and other values same as example 1, the results are t_1 = 2.11 weeks, T = 5.12 weeks, I_1 = 19.8 units and minimum C = \$ 165.11.
5. For C_o = \$180/set up and other values same as example 1, the results are t_1 = 1.76 weeks, T = 4.33 weeks, I_1 = 16.69 units and minimum C = \$ 178.56.
6. For C_h = \$ 2 per unit per week and other values same as example 1, the results are t_1 = 2.72 weeks, T = 6.44 weeks, I_1 = 25.07 units and minimum C = \$ 169.88.
7. For d = 8 units/week, R = 20 units/week and other values same as example 1, the results are t_1 = 1.92 weeks, T = 4.44 weeks, I_1 = 21.75 units and minimum C = \$ 235.64.
8. For R = 20 units/week, C_o = \$180/set up and other values same as example 1, the results are t_1 = 1.32 weeks, T = 4.04 weeks, I_1 = 17.76 units and minimum C = \$ 183.88.
9. For d = 8 units/week, R = 20 units/week, C_o = \$180/set up and other values same as ex. 1, the results are t_1 = 1.65 weeks, T = 3.85 weeks, I_1 = 18.82 units and minimum C = \$ 221.15.
10. Taking different values of H and constant values of other parameters same as in example 1, the results are shown in the table (1) and figures (3), (4) and (5). The Karl Pearson's coefficient of correlation for the time of production, the time of one cycle, the average total cost per unit time and the maximum inventory level with the hazardous rate per unit time is shown in the table 2.

Table 1

C_o	C_h	C_p	R	d	H	t_1	T	C(T)	I_1
240	4	16	16	6	0.04	2.07	5.18	188.07	19.88
240	4	16	16	6	0.06	2.05	4.99	191.43	19.30
240	4	16	16	6	0.08	2.03	4.83	194.66	18.78
240	4	16	16	6	0.10	2.02	4.68	197.77	18.29
240	4	16	16	6	0.12	2.01	4.55	200.76	17.84
240	4	16	16	6	0.14	2.00	4.44	203.65	17.43
240	4	16	16	6	0.16	1.99	4.33	206.45	17.04
240	4	16	16	6	0.18	1.98	4.24	209.15	16.68
240	4	16	16	6	0.20	1.98	4.15	211.77	16.34

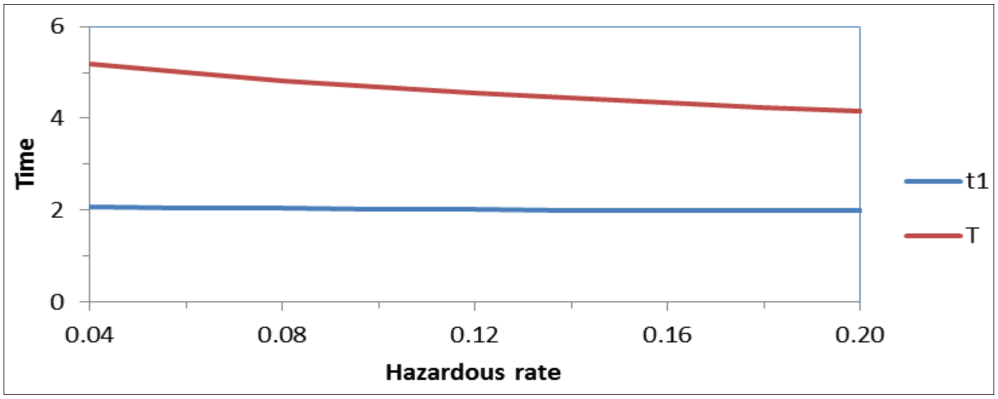

Fig 3

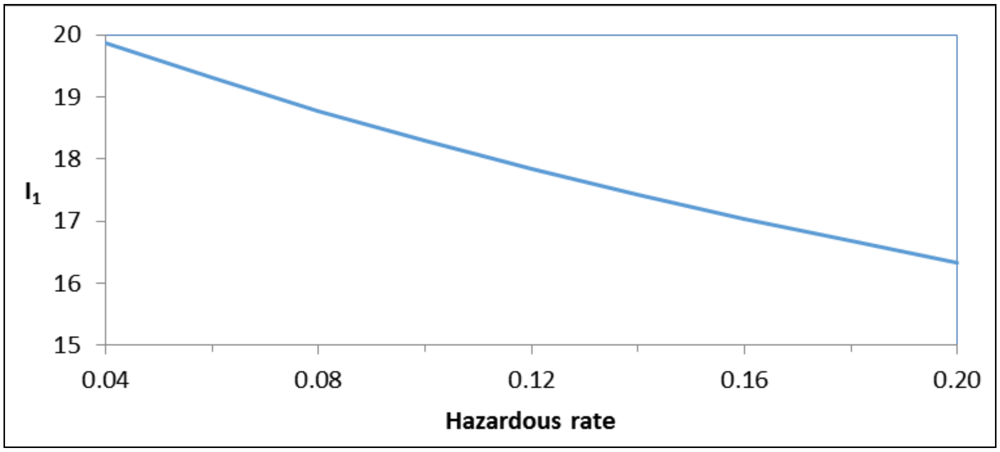

Fig 4

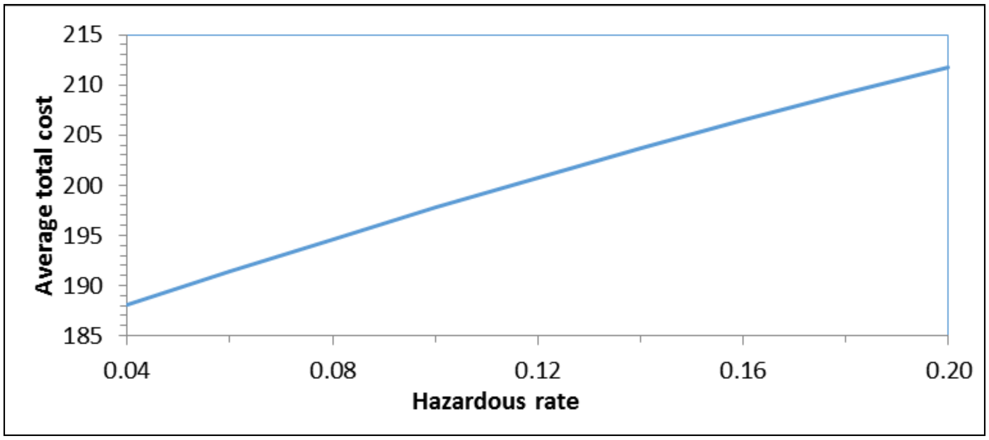

Fig 5

Table 2

Coefficient of Correlation	t_1	T	C(T)	I_1
H	-0.9779893	-0.99197134	0.99918517	-0.996283

5. Conclusion

In this paper a correlation analysis is presented between hazardous rate per unit time of two-parameter negative exponential distribution and various variables. Keeping constant all the parameters like inventory holding cost per unit time, production cost of one item, production rate per unit time and rate of demand, hazardous rate has a negative high correlation with the time of production, the time of one cycle and the maximum inventory level but it has a positive high correlation with the average total cost per unit time. Hence we conclude that the average total cost increases highly with hazardous rate. Therefore it should be controlled for maximization of profit.

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