



## Pulsatile flow of non-newtonian blood through multiple stenosed artery

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### Abstract

Pulsatile flow of blood under the influence of externally imposed magnetic field and periodic body acceleration through a multistenosed artery is studied. In this problem a Mathematical model is developed by treating blood as a non Newtonian fluid. The pulsatile is analyzed by considering a periodic pressure gradient which is a function of time. In this analysis we try to find the computational result by using the Perturbation analysis, assuming that the Womersley frequency parameter is very small. Variation of mean flow rate against Reynolds number for wave number and back pressure, Variation of mean flow rate against amplitude ratio for Reynolds numbers and back pressure and Velocity profiles along a vertical line in a time period have been discussed with the help of graphs.

**Keywords:** pulsatile flow, stenosed artery, perturbation analysis

### 1. Introduction

In general, peristaltic pumping is characterized by the dynamic interaction of fluid flow with the movement of a flexible boundary. The fluid mechanics of peristaltic pumping have been studied by Jaffrin and Shapiro [2]. It is considered that the length of the peristaltic wave is large compared with the channel width and the appropriate Reynolds number is sufficiently small for the flow to be considered inertia-free. The increase of drag due to the pseudoplastic anomaly is more significant at large viscosity ratio parameter. Approximate expressions of the drag coefficient of a non-Newtonian drop moving in a power-law fluid at the intermediate Reynolds number were presented by Nakano and Tien [1]. The effect of viscoelasticity on the drag of a drop in a non-Newtonian fluid system was theoretically analyzed by Wagner and Slattery [14] using perturbation techniques: peristalsis is an inherent property of many biological systems having smooth muscle tubes which transport biofluids by its propulsive movements. Peristaltic flow of a second-order fluid in a planar channel and in an axi-symmetric tube is studied by Siddiqui *et al.* [12] under long wavelength assumption.

Peristaltic pumping is a form of material transport that occurs when a progressive wave of area contraction or expansion propagates along the length of a distensible tube containing some material. The effect of a magnetic field on the peristaltic transport of blood in a non-uniform two-dimensional channel is given by Mekheimer [7]. Here blood is represented by a couple stress fluid (which is special case of a non-Newtonian fluid and intended to take into account the particle size effect). It has been pointed out that surface-active agents play an important role in the motion of fluid spheres moving through non-Newtonian fluids. Peristaltic pumping of viscoelastic fluids was investigated by Bohme and Friedrich [15]. Shehawey and Mekheimer [8] discussed couple stresses in peristaltic transport of fluids. Peristaltic transport of a physiological fluid in a tapered channel is studied by Eytan *et al.* [9] and this model is applied to explain embryo transport with uterine cavity. Moreover, Vajravelu and Sreenadh [5] investigated the peristaltic transport of a Herschel-Bulkley fluid in an inclined tube.

Motivated by these studies, the peristaltic flow of a non-Newtonian fluid in a tube and the effect of velocity distribution are investigated. Some deductions are made and results are found to be in agreement with available experimental data.

### 2. Mathematical Model

In The construction of the multiple stenosed artery in non dimensional form is given by

$$R(z) = \begin{cases} 1 - A \left[ L_s^{\Gamma-1} \{z - (b_1 + b_2 + b_3)\} - \{z - (b_1 + b_2 + b_3)\} \right] & \text{for } (b_1 + b_2) \leq z \leq (b_1 + b_2) + L_s \\ 1 & \text{otherwise} \end{cases} \quad (1)$$

Where  $A = \frac{\delta}{R_0(L_s)^\Gamma}$  (2)

Where  $\delta$  is maximum height of stenosis.

$$z = (b_1 + b_2 + b_3) + \frac{L_s}{\Gamma^{\Gamma-1}} \quad (3)$$

Where  $\Gamma \geq 2$  is the parameter for determining the shape of stenosis.  $\frac{\delta}{R_0} \ll 1$ .

The specified momentum equation for the flow in cylindrical coordinate system is

$$\rho \frac{\partial u'}{\partial t'} = -\frac{\partial p'}{\partial z'} - \frac{1}{r'} \frac{\partial(r' \tau')}{\partial r'} + G(t') + \mu_0 M \frac{\partial H'}{\partial z'} \quad (4)$$

The above assumptions for Navier –Stokes equations is given by is magnetic permeability,  $M$  is magnetization,  $H'$  is magnetic field density and is pressure density of the fluid and is the periodic body acceleration which is in the axial direction then is given by

$$H(t) = a_0 \cos(\omega_1 t + \theta) \quad t' \geq 0 \quad (5)$$

Where  $a_0$  is its amplitude, where  $f$  is frequency and  $\theta$  is its phase difference. The frequency of body acceleration is assumed to be small so that wave effects can be neglected. The pressure gradient is taken as:

$$-\frac{\partial p'}{\partial z'} = A_0 + A_1 \cos(\omega_2 t + \theta) \quad t \geq 0 \quad (6)$$

Where  $A_0$  is the steady state part of pressure gradient,  $A_1$  is the amplitude of the oscillatory part  $f_2$  being the heart pulse frequency. For casson fluid, the relation between shear stress and shear rate is:

$$-\frac{\partial u'}{\partial r'} = \frac{(\sqrt{\tau'} - \sqrt{\tau_0'})^2}{\mu} \quad \text{for } \tau' \leq \tau_0' \quad (7(a))$$

$$\frac{\partial u'}{\partial r'} = 0, \quad \text{for } \tau' \geq \tau_0' \quad (7(b))$$

The boundary conditions governing the problem as follows:

$$u' = 0 \text{ at } r' = R'(z') \quad (8(a))$$

$$\tau' \text{ is finite at } r' = 0 \quad (8(b))$$

$$u' = u_0' \text{ at } r' = R_0' \quad (8(c))$$

Where  $u_0'$  is the core Velocity.

$$z = \frac{z'}{R_0'}, \quad u = \frac{u'}{A_0 R_0'^2}, \quad R(z) = \frac{R'(z)}{R_0'}, \quad r = \frac{r'}{R_0'}, \quad t = \omega_2 t', \quad \delta = \frac{\delta'}{R_0'} \quad (9)$$

$$\tau = \frac{2\tau'}{A_0 R_0'}, \quad \theta = \frac{2\tau_0'}{A_0 R_0'}, \quad \omega = \frac{\omega_1}{\omega_2}, \quad B = \frac{a_0}{A_0}, \quad e = \frac{A_1}{A_0}, \quad \alpha^2 = \frac{R_0'^2 \omega_2 \rho}{\mu}, \quad H = \frac{H'}{H_0}$$

Where  $H_0$  is transverse uniform magnetic field the non- dimensional momentum equation (4) becomes

$$\alpha^2 \frac{\partial u}{\partial t} = 4(1 + e \cos t) + 4B \cos(\omega t + \theta) + \frac{2}{r} \frac{\partial(r \tau)}{\partial r} + F_1 \frac{\partial H}{\partial z} \quad (10)$$

Where  $\alpha^2 = \frac{R_0^2 \omega_2 \rho}{\mu}$  and  $F_1 = \frac{\mu_0 H_0 M}{\frac{A_0 R_0^2}{4}}$

Equation (7(a) and 7(b) can be written as

$$\tau^{1/2} = \theta^{1/2} + \frac{1}{\sqrt{2}} \left( -\frac{\partial u}{\partial r} \right)^{1/2} \quad \text{if } \tau \geq \theta \quad (11(a))$$

$$\frac{\partial u}{\partial r} = 0 \quad \text{if } \tau \leq \theta \quad (11(b))$$

The boundary conditions 8(a), 8(b) and 8(c) reduced to

$$u = 0 \quad \text{at } r = R(z) \quad (12(a))$$

$$\tau \text{ is finite at } r = 0 \quad (12(b))$$

$$u = u_0 \quad \text{at } r = R_0 \quad (12(c))$$

### 3. Analytical solution of the problem

Considering the Womersley parameter to be small, the velocity  $u$ , shear stress plug core radius  $R_c$  and plug core velocity  $u_c$  are expressed in the following form:

$$u(z, r, t) = u_0(z, r, t) + \alpha^2 u_1(z, r, t) + \dots \quad (13(a))$$

$$\tau(z, r, t) = \tau_0(z, r, t) + \alpha^2 \tau_1(z, r, t) + \dots \quad (13(b))$$

$$R_c(z, r, t) = R_{0c}(z, r, t) + \alpha^2 R_{1c}(z, r, t) + \dots \quad (13(c))$$

$$u_c(z, r, t) = u_{0c}(z, r, t) + \alpha^2 u_{1c}(z, r, t) + \dots \quad (13(d))$$

Substituting (13a) and (13b) in equation (10) and equating constant terms we get

$$\frac{\partial}{\partial r}(r\tau_0) = -2r \left[ (1 + e \cos t) + B \cos(\omega t + \theta) + F_1 \frac{\partial H}{\partial z} \right] \quad (13(e))$$

$$\frac{\partial}{\partial r}(r\tau_0) = -2r \left[ g(t) + F_1 \frac{\partial H}{\partial z} \right] \quad (14)$$

$$\frac{\partial}{\partial t}(u_0) = -\frac{2}{r} \frac{\partial}{\partial r}(r\tau) \quad (15)$$

Where  $g(t) = (1 + e \cos t) + B \cos(\omega t + \theta)$  (15(a))

Integrating equation (14) and using boundary conditions 8(a), 8(b) and 8(c) we get

$$\tau_0 = -r \left[ g(t) + \frac{F_1}{4} \frac{\partial H}{\partial z} \right] \quad (15(b))$$

Substituting (13a) and (13b) in equation (11a) then we get

$$-\frac{\partial}{\partial r}(u_0) = 2 \left[ \theta + \tau_0 - 2\sqrt{\theta\tau_0} \right] \quad (16)$$

$$-\frac{\partial}{\partial r}(u_1) = 2\tau_1 \left[ 1 - \sqrt{\frac{\theta}{\tau_0}} \right] \quad (17)$$

Using relation (15a) and (15b) and boundary condition (8a) and (8b) in equation (16) then we obtain the axial velocity in equation (18)

$$u_0 = \left[ \left( g(t) + \frac{F_1}{4} \frac{\partial H}{\partial z} \right) (R^2 - r^2) - \frac{8 \sqrt{\theta \left( g(t) + \frac{F_1}{4} \frac{\partial H}{\partial z} \right)}}{3} \left( R^{\frac{3}{2}} - r^{\frac{3}{2}} \right) + 2\theta(R-r) \right] \tag{18}$$

The core velocity can be determined by equation (18):

$$u_{0c} = \left[ \left( g(t) + \frac{F_1}{4} \frac{\partial H}{\partial z} \right) (R^2 - R_{0c}^2) - \frac{8 \sqrt{\theta \left( g(t) + \frac{F_1}{4} \frac{\partial H}{\partial z} \right)}}{3} \left( R^{\frac{3}{2}} - R_{0c}^{\frac{3}{2}} \right) + 2\theta(R - R_{0c}) \right] \tag{19}$$

By using equation (15),(18), and (19) as:

$$\tau_1 = \frac{g'(t)R^3}{8} \left[ 2 \left( \frac{r}{R} \right) - \left( \frac{r}{R} \right)^3 - \frac{8}{21} \frac{\sqrt{\theta}}{\sqrt{\left( g(t) + \frac{F_1}{4} \frac{\partial H}{\partial z} \right) R}} \left\{ 7 \left( \frac{r}{R} \right) - 4 \left( \frac{r}{R} \right)^{\frac{5}{2}} \right\} \right] \tag{20}$$

$$u_1 = \frac{g'(t)R^4}{8} \left[ \left( \frac{r}{R} \right)^4 + 4 \left( \frac{r}{R} \right)^2 + 3 + \frac{\sqrt{\theta}}{\sqrt{\left( g(t) + \frac{F_1}{4} \frac{\partial H}{\partial z} \right) R}} \left\{ \frac{16}{3} \left( \frac{r}{R} \right)^2 - \frac{424}{147} \left( \frac{r}{R} \right)^{\frac{5}{2}} \right\} - \frac{1144}{147} + \frac{\theta}{\left( g(t) + \frac{F_1}{4} \frac{\partial H}{\partial z} \right) R} \left\{ \frac{128}{63} \left( \frac{r}{R} \right)^3 - \frac{64}{9} \left( \frac{r}{R} \right)^{\frac{5}{2}} + \frac{320}{63} \right\} \right] \tag{21}$$

$$u_{1c} = \frac{g'(t)R^4}{8} \left[ \left( \frac{R_{0c}}{R} \right)^4 + 4 \left( \frac{R_{0c}}{R} \right)^2 + 3 + \frac{\sqrt{\theta}}{\sqrt{\left( g(t) + \frac{F_1}{4} \frac{\partial H}{\partial z} \right) R}} \left\{ \frac{16}{3} \left( \frac{R_{0c}}{R} \right)^2 - \frac{424}{147} \left( \frac{R_{0c}}{R} \right)^{\frac{5}{2}} \right\} - \frac{1144}{147} + \frac{\theta}{\left( g(t) + \frac{F_1}{4} \frac{\partial H}{\partial z} \right) R} \left\{ \frac{128}{63} \left( \frac{R_{0c}}{R} \right)^3 - \frac{64}{9} \left( \frac{R_{0c}}{R} \right)^{\frac{5}{2}} + \frac{320}{63} \right\} \right] \tag{22}$$

$$u = \left[ \left( g(t) + \frac{F_1}{4} \frac{\partial H}{\partial z} \right) (R^2 - r^2) - \frac{8 \sqrt{\theta \left( g(t) + \frac{F_1}{4} \frac{\partial H}{\partial z} \right)}}{3} \left( R^{\frac{3}{2}} - r^{\frac{3}{2}} \right) + 2\theta(R-r) \right] + \alpha^2 \frac{g'(t)R^4}{8} \left[ \left( \frac{r}{R} \right)^4 + 4 \left( \frac{r}{R} \right)^2 + 3 + \frac{\sqrt{\theta}}{\sqrt{\left( g(t) + \frac{F_1}{4} \frac{\partial H}{\partial z} \right) R}} \left\{ \frac{16}{3} \left( \frac{r}{R} \right)^2 - \frac{424}{147} \left( \frac{r}{R} \right)^{\frac{5}{2}} \right\} - \frac{1144}{147} + \frac{\theta}{\left( g(t) + \frac{F_1}{4} \frac{\partial H}{\partial z} \right) R} \left\{ \frac{128}{63} \left( \frac{r}{R} \right)^3 - \frac{64}{9} \left( \frac{r}{R} \right)^{\frac{5}{2}} + \frac{320}{63} \right\} \right] \tag{23}$$

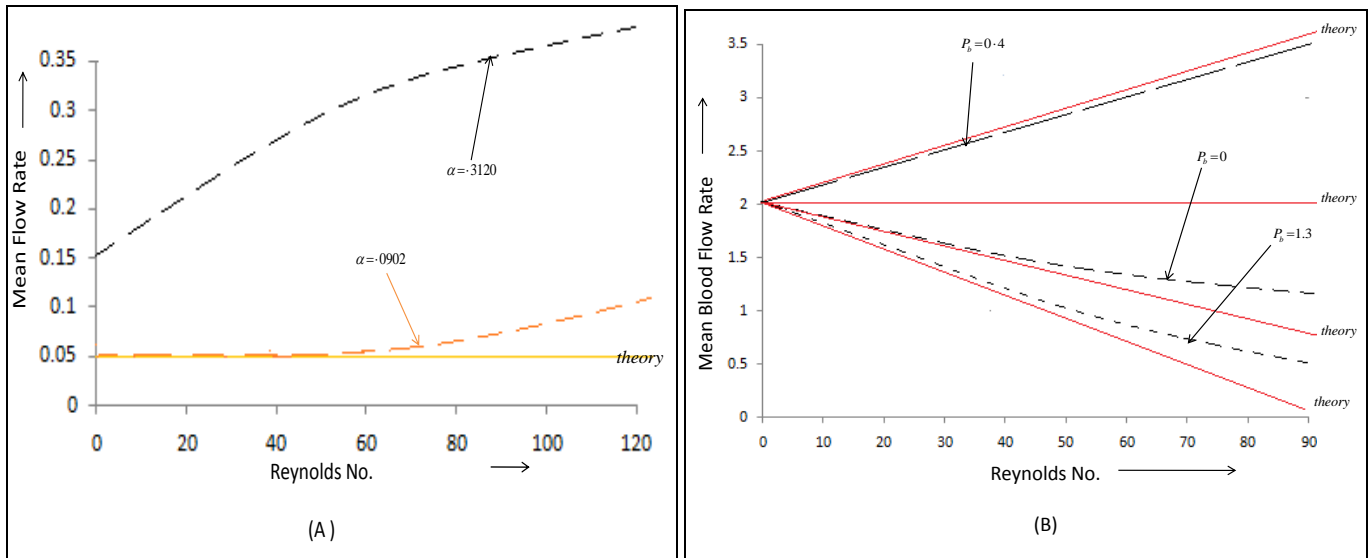
The volumetric flow rate Q(t) is given by

$$Q(t) = 4 \int_0^R u r dr \tag{24}$$

$$Q(t) = R^4 \left\{ \left( g(t) + \frac{F_1}{4} \frac{\partial H}{\partial z} \right) \frac{1}{4} + \frac{4 \sqrt{\theta \left( g(t) + \frac{F_1}{4} \frac{\partial H}{\partial z} \right)}}{7} + \frac{1}{3} \frac{\theta}{R} + \alpha^2 \frac{CR^2}{16} \left[ \frac{2}{3} + \frac{120 \sqrt{\theta \left( g(t) + \frac{F_1}{4} \frac{\partial H}{\partial z} \right)}}{77 \sqrt{R}} + \frac{32}{35} \frac{\theta}{R} \right] \right\}$$

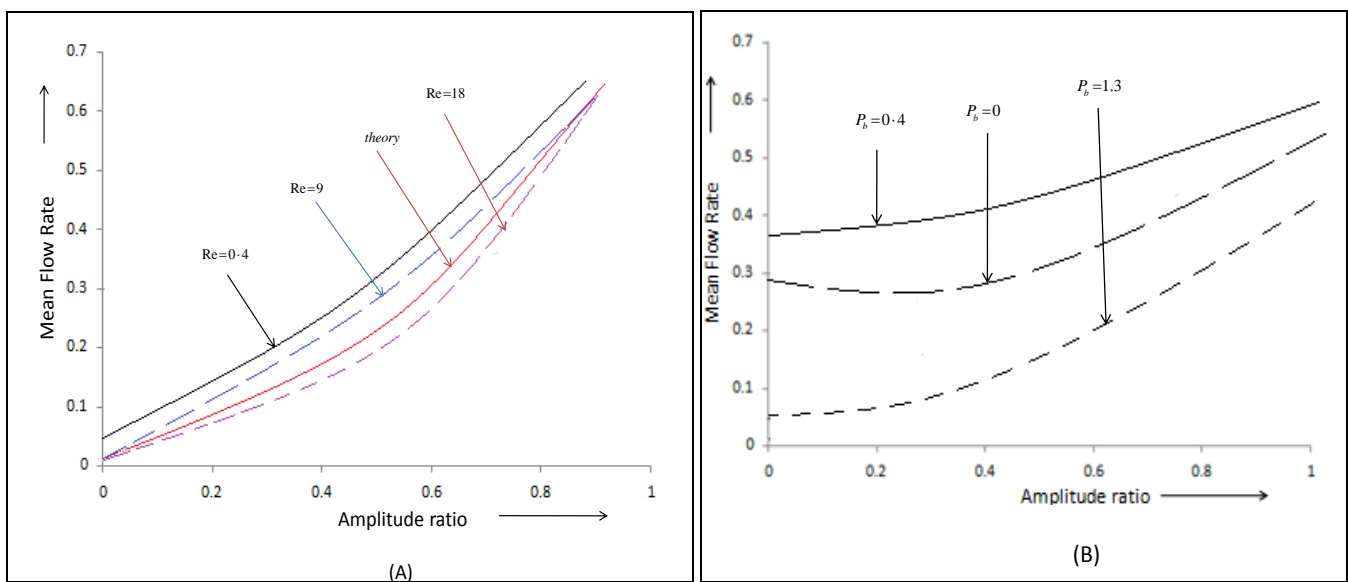
### 4. Result and Discussion

Figure 1 confirmed the variant of average flow rate in opposition to Reynolds number for amplitude ratio  $\varepsilon=0.2$  and back pressure  $P_b=0$ . The Reynolds number is fluctuated up to 100. Two wave numbers are considered for low wave number case ( $\alpha=0.0902$ ) to 0.3120, the blood flow rate is deeply increased and turn into growing function of Reynolds number.

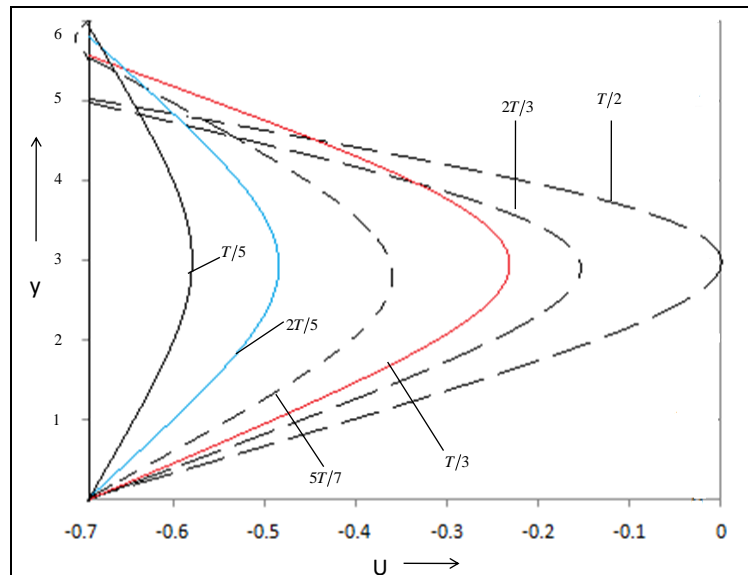


**Fig 1:** Variation of mean flow rate against Reynolds number with  $\varepsilon=0.2$  for (a) two wave numbers  $P_b=0$  and (b) different back pressures  $\alpha=0.0902$

The consequences of amplitude ratio on arithmetic blood flowing rate for the case of zero back blood pressure and wave number  $\alpha=0.0902$  are demonstrated in Fig. 2(a). It is observed from Fig. 2(a) that this equation is applicable for the Reynolds numbers up to 20. Figure 2(b) showed the outputs for back pressures -0.5, 0, and 1.5. At minimum amplitudes, the blood flow in the system is primarily driven by the blood pressure gradient. As an output, the variation between these different back pressure cases is important at low principles of amplitude. Still, the differences diminish as  $\varepsilon$  increases for the reason that amplitude result slowly becomes more significant. When the system is entirely obstruct at  $\varepsilon=0.2$  and  $Q=2$ , it's a conclusion why all the curves lean to merge at high amplitudes.



**Fig 2: a.** Variation of mean flow rate against amplitude ratio with  $\alpha=0.0902$  for, (a) different Reynolds numbers  $P_b=0$  and (b) different back pressures  $Re=18$



**Fig 2:** Velocity (U) profiles along a vertical line in a time period

## 5. Conclusion

Due to the complicated nature of the peristaltic flow, a small number of works have been done in this field. The method used in this study provides a quite successful choice for dealing with such problems. This method provides exact solutions in the peristaltic flow without any assumed restrictions on the actual physical quantities that appear in the governing equations of the problem considered. Important phenomena have observed from our calculation results.

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