



Analytical study of wave-break in the vacuum-plasma interface during the interaction of an intense laser pulse

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Abstract

In this paper, the wave soften up the plasma-vacuum interface amid the extreme laser connection is researched. Since the nonlinear wave breaking is a non-adiabatic process, the completely active 1D-3V Particle-In-Cell (PIC) recreation tests are performed to recognize whether that the cause of this system is electromagnetic or electrostatic. Our reenactment comes about demonstrate that the nonlinear wave breaking on the vacuum-plasma interface has electrostatic source. Also, it is discovered that for beat lengths surpassing the plasma wavelength this electrostatic marvel comes in conjunction with some dynamic electromagnetic impacts having a similar effect on the electron increasing speed. In such matters, we direct modern reenactments confining these electromagnetic impacts and concentrate the impacts of the beat parameters, for example, the beat rise time, beat length, and heartbeat shape on the limit nonlinear wave breaking. The investigation of the beat rise-time variety impacts demonstrates that as the ascent time of the laser beat diminishes, the quantity of the electrons associated with the nonlinear wave breaking, most extreme vitality of the caught electrons and the way length of the quickened electrons in the stage space are expanded. Additionally, the investigation of stage space and field designs in our reenactment shows that the lessening of the beat level best length time causes that the littler piece of the electrons and the littler bit of the wake wave include in the nonlinear wave breaking.

Keywords: vacuum-plasma, laser pulse, electromagnetic

1. Introduction

The communication of a ultra-short extreme laser beat with the under-thick plasma is a standout amongst the most inspiring subjects in the plasma material science. In this specific circumstance, countless molecule marvels turn out to be especially intriguing when the powerful laser causes the electron tremble speed to wind up exceptionally relativistic. Among these marvels, the wake field's generation [1-6], scrambling and balance of the laser pulse [7-10], stochastic movement of the electrons [11-15] and wave breaking processes [16-19] have pulled in developing enthusiasm because of their fundamental part in the idea of laser-plasma quickening agent. The last case, the wave breaking impact, is a standout amongst the most essential wonders in the laser-plasma idea and it is the premise of some turbulent movement of electrons and furthermore increasing speed components, for example, quickening started by the liquid wave breaking, coordinate laser speeding up, and quickening began from the nonlinear wave breaking by means of the plasma-vacuum limit effect [14, 15, 20].

Dawson has explored the longitudinal wave softening up the cool plasma in one dimensional case for the principal time [21]. As specified in the Dawson' hypothesis, for both relativistic and nonrelativistic plasmas, when the pinnacle of the liquid speed of the swaying electrons is equivalent to the stage speed of the plasma wave, the longitudinal wave softening is happened up the frosty plasma. Truth be told, the wave breaking peculiarity shows up when the Jacobian of the change from the Eulerian to the Lagrangian facilitate vanishes. In one measurement, this peculiarity can be reintroduced by

$v_q = v_{ph}$, where v_q is the electron tremble speed and v_{ph} is the wave stage speed. This condition happens when the greatest adequacy of the wavering wake (Langmuir wake) surpasses or levels with the converse of the wave-number in the direct administration. The above measure on the most extreme sufficiency of the swaying wake is known as 1D icy liquid wave breaking edge. In the idea of the semi static estimate, there are some inquires about which explored the nonlinear collaboration of the exceptional laser beat with plasmas and the beat spread properties through the under-thick plasmas in one and two dimensions [1, 22]. Mori and his colleagues researched the impact of Raman forward disseminating on proliferation and connection of short laser beat with the under-thick plasmas [7]. However, in the greater part of the things specified in the writing, the wave breaking consequences for the plasma vacuum interface have been ignored.

The nonlinear wave breaking properties has been essentially examined in the before literature [23, 24]. In this unique situation, Akhiezer and Polovin set up the imperative on the abundancy of the stationary nonlinear Langmuir waves [25]. In an inhomogeneous plasma, the wave number of the longitudinal Longmuir wave (ω, k) relies upon time through the relation [23].

$$\partial k / \partial t = -\partial \omega / \partial x, \quad (1)$$

Where ω and k are the wave number and recurrence of the wave, individually. For the longitudinal Langmuir wave, we have $\omega \cong \omega_p = (4\pi n_0 e^2 / m_e)^{1/2}$ in this way, the presence of a

spatial slope in the plasma thickness can prompt the development of the wave number with the time. This development brings about breaking of the wave, regardless of whether at first the wave plentifulness is beneath the wave breaking edge. In this condition, just a little bit of the wave is associated with the wave breaking and the wave softening happens locally up the Langmuir waves [20].

Bulanov and his associates examined the excitation of a relativistic Longmuir wave and nonlinear wave breaking of the Longmuir wave for a precarious and smooth plasma boundary [26]. Also, the utilization of nonlinear wake wave breaking for the infusion of electrons into the wave quickening stage is investigated [20]. The recreation investigation of the impact of the nonlinear wave softening up vacuum plasma limit on the electron increasing speed system in the stochastic way and vitality dissemination capacity of the quickened electrons has been accounted for in our past works [14, 15].

In light of the non-adiabatic nature of the nonlinear wave breaking at the vacuum plasma limit, the electrostatic characters of this marvel can't be considered as far as the semi static estimation. In this manner, the starting point of the nonlinear wave softening is disregarded up the past writing. Without a doubt, as an issue of rule, is the source of this marvel the electrostatic or electromagnetic? Also, for beat lengths surpassing the plasma wavelength the nonlinear wave softening impact comes up conjunction with some dynamic electromagnetic impacts having a similar effect on the electron speeding up. What's more, this issue isn't considered in the past writing. In such matters, we lead refined reenactments secluding the electromagnetic impacts and concentrate the impacts of the beat parameters, for example, the beat rise time, beat length, and heartbeat shape on the limit nonlinear wave breaking.

In alternate words, how variety of the ponderomotive power because of the ascent of the beat in the short ascent time beat case, and electromagnetic impacts, for example, dispersing and modulational hazards of the wave bundle in the long length beat case can influence the wake wave and wave breaking on the vacuum-plasma limit? In this work, we attempt to answer these inquiries by utilizing completely dynamic 1D-3V PIC reproduction tests.

2. Simulation Results and Discussion

The reproduction parameters through this segment, the reenactment parameters are abridged here. These reproductions are led by a 1d-3v (one spatial and three speed measurements) pic-code composed by j. yazdanpanah [27, 28]. The laser wavelength has been invariantly set to $\lambda = 1\mu\text{m}$ in all cases. For all run-occasions, we have utilized hydrogen plasma with beginning advance like profile. high spatial and stage space resolutions have been utilized as a part of recreations by letting 200 cells for every laser wavelength and no less than 64 particles for each a solitary cell. The spatial determination ensures the plasma against the un-physical warming created by the limited network flimsiness by fitting determination of the length λd , i.e. $dx/\lambda d \cong 0.3$. The extent of the recreation box is 600λ with open and intelligent limit conditions being connected to its finishes for the fields and particles, individually. The standardized laser plenty fullness

and the thickness of plasma are individually settled to $a_0 = 2$ and $n = 0.02n_{cr}$. Furthermore, in the all reenactment plots the plasma begins from $x = 90\mu\text{m}$. The beat envelope comprises of three sections: level best focal part sided by two sinusoidal-formed rising and falling parts.

We play out our image recreations in two distinctive electrostatic (es) and the typical completely electromagnetic (em) modes. In the es mode the plasma optical reaction is falsely disposed of by disregarding the transverse plasma-current through Maxwell conditions while fusing the nonlinear gathering speed in these conditions as indicated by the connection

$$v_g = c\sqrt{1 - \gamma_L n_{e0}/n_{cr}} \quad \left(\gamma_L = \sqrt{1 + a_0^2/2} \right)$$

Given by akhiezer and polovin [25]. The movement conditions and the poisson condition are dealt with as in the typical em show. With these contemplations, the light spreads unaffected inside the plasma (dissipating and tweaks are forestalled) prompting an enduring wake field excitation. What's more, in light of the fact that the beat bunch speed is not quite the same as c , the electron catching in the plasma wave is permitted as in the standard em show., to begin with, the connection of a limited laser beat length 260 fs, with the short ascent/fall time-term 30 fs, and long level best span 200 fs is examined. This investigation is performed in two distinct conditions, under electromagnetic condition in which, the beat development impacts as the diffusing and adjustment impacts is viewed as and furthermore under electrostatic condition, barring the beat tweak and the scrambling impacts. What's more, for each case, the adiabatic arrangements (semi static Maxwell-fluid condition) and direct pic recreation comes about (with any guess) are exhibited. fig. 1 shows adiabatic arrangements and direct pic recreation consequences of the worldly development of the longitudinal electric field and transverse vector potential in boards [(a)- (c)] (left) and the stage space of the electrons in boards [(d)- (f)] (appropriate), under electromagnetic condition at various circumstances $t = 252$ fs [(a) and (d)], $t = 360$ fs [(b) and (e)], and $t = 500$ fs [(c) and (f)] ($1\text{fs} = 10^{-15}\text{s}$). As indicated by these figures, two wave softening components are seen up the two electrons stage space and development examples of longitudinal and transverse fields. To begin with, nonlinear wave breaking of the wake wave locally happens on the vacuum-plasma limit. The stochastic example in the electron stage space is clear in this area, and the discharged electrons, because of the nonlinear wave breaking, are privately created. It is clear from the reenactment comes about that exclusive a little segment of the wave and a minor piece of the electrons are associated with the wave breaking. It merits saying that, this system is prevailing at the beginning time of the connection. The second wave softening system can be seen up figs. 1 as it turned up in the long time is wave breaking of the longitudinal space charge wake wave toward the finish of the beat wave parcel because of the beat balance. As opposed to the principal wave breaking component, the adiabatic arrangements are steady with the pic reenactment result in the beat body before happening of the breaking system and an expansive segment of the longitudinal wave (wake wave) and a substantial piece

of the electrons are engaged with this wave breaking instrument.

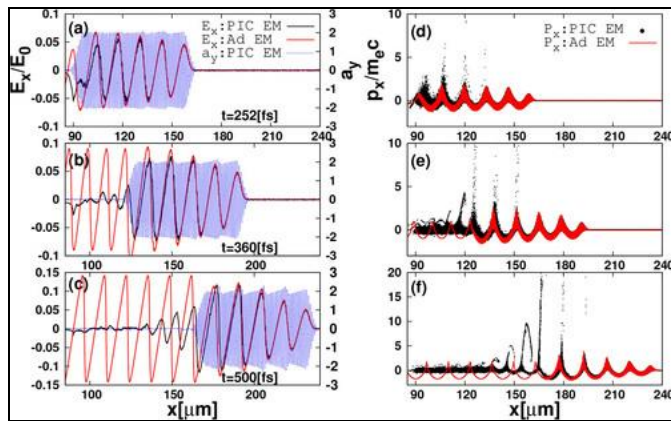


Fig 1: The adiabatic solutions and direct PIC simulation results of the temporal evolution of the longitudinal electric field and transverse vector potential in panels [(a)-(c)] (left) and the phase space of the electrons in panels [(d)-(f)] (right), under electromagnetic condition at different times $t = 252$ fs [(a) and (d)], $t = 360$ fs [(b) and (e)], and $t = 500$ fs [(c) and (f)].

In any case, the inception of the nonlinear wave breaking on the vacuum-plasma limit is not perceived by the above recreation analyze. As indicated by Fig. 1, the wave breaking of the longitudinal space charge wake wave in the body of the plasma and toward the finish of the laser heartbeat can be begun from the tweak, and mutilation and development of the beat by some electromagnetic instruments, for example, the diffusing of the beat by particles bundle in the plasma. To decide the starting point of the vacuum-plasma limit wave breaking system, the electromagnetic marvel of the communication, for example, adjustment and dissipating of the beat are killed in Fig. 2. The adiabatic arrangements and direct PIC reproduction consequences of the transient advancement of the longitudinal electric field and transverse vector potential in boards [(a)- (c)] (left) and stage space of electrons in boards [(d)- (f)] (appropriate), under the considered electrostatic condition, with an indistinguishable parameters from in Fig. 1 are outlined in Fig. 2. As indicated by this figure, the time advancement of the longitudinal electric field demonstrates that there is a contrast between the adiabatic arrangement (QSA liquid arrangement) and the PIC reproduction comes about along the vacuum-plasma limit. Clearly the normal wavering example of the longitudinal electric wave is wrecked, and thus the wave softening of the wake wave up the cooperation locale happens. Additionally, the stage space of the electrons exhibit a stochastic example for the electron movements in this area, locally. Plainly a little segment of the wave and just a few electrons are associated with this wave breaking. Be that as it may, some of electrons discharged because of this nonlinear wave breaking component can be quickened by the wake wave over the time. As per Figs. 2(a) – 2(c), the nonlinear wave breaking impacts are diminished over the time as the beat spreads in the plasma. It is noticed that the QSA arrangement and PIC reenactment comes about are coordinated in the body of the beat. Furthermore, the normal wavering of the longitudinal space

charge wave in the body of the beat isn't bothered. Therefore, the nonlinear wave breaking of the wake wave at the vacuum-plasma interface has totally electrostatic root. In such manner, a correlation of the outcomes in Figs. 1 and 2, in the nearness and nonappearance of the electromagnetic impacts, demonstrates that the nonlinear wave breaking on the vacuum-plasma interface is the overwhelming system at the prior circumstances. In the both purpose of perspectives, stage space and fields designs show comparable advancements for the electrons, wake wave and fields at the prior circumstances roughly.

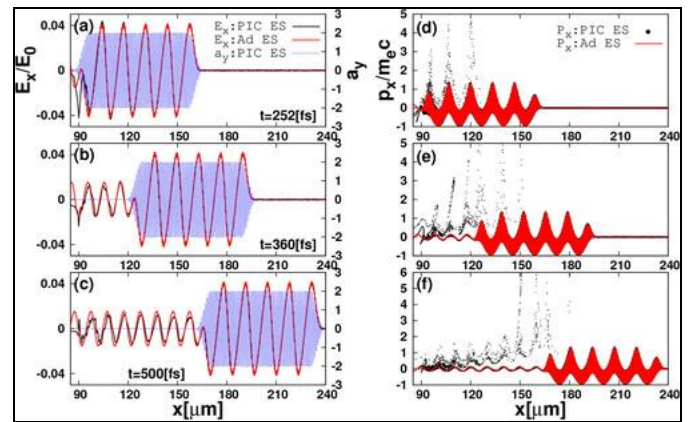


Fig 2: The adiabatic solutions and direct PIC simulation results of the temporal evolution of the longitudinal electric field and transverse vector potential in panels [(a)-(c)] (left) and phase space of electrons in panels [(d)-(f)] (right), under electrostatic condition, at different times $t = 252$ fs [(a) and (d)], $t = 360$ fs [(b) and (e)], and $t = 500$ fs [(c) and (f)].

To demonstrate the distinctions of the vitality conveyance work under electromagnetic and electrostatic conditions, the vitality range is shown in detail in Figs. 3(a) and 3(b), separately. Obviously, the distinctions of the vitality circulation work in these two cases are numb at the early circumstances of collaboration. Actually, in this area the discharged electrons because of the nonlinear wave softening are caught up the wake field and quickened. In the two cases, they assume an imperative part in the arrangement of disorganized example in the stage space in this district, locally. These impacts are reliable with the past literature [20, 26]. On the other hand, as per Fig. 3, the time advancement of the vitality appropriation work in these two cases will be distinctive as the beat engenders through the plasma. As indicated by Fig. 1, when the beat moves from the plasma limit after some time, the electromagnetic impacts, for example, the diffusing and tweak of the laser beat show up in the stage space and fields designs. At this stage, the diffusing and the tweak of the laser heartbeat can infer a ponderomotive power with a similar period of the longitudinal space charge wave [7, 29]. Therefore, this component can open up the space charge wave abundancy. The sufficiency of the space charge wave, wake wave, grows up to the liquid wave breaking limit level toward the finish of the laser beat wave bundle over the time [15, 21]. These impacts saw in the stage space design, are related with the age of the high quickened electrons and confused examples in the electromagnetic approach. Another

electromagnetic wonder which could occur in the body of the laser beat is age of the electrons with stochastic movement within the sight of two contra-proliferate electromagnetic waves [11, 12, 14, 30]. In the cooperation of the laser beat with long level best by plasma, for example, considered for the situation (200 fs level best time), the scattered and the occurrence waves assume the part of two contra-spread waves. As per the Medonca criteria, if the adequacy of these two waves surpasses the edge $\alpha_{1a2} \geq 1/16$, the Lagrangian of the electron in the fields of these waves moves toward becoming non-integrable.³⁰ Therefore, the consistent electron movement separates and the movement of the electron winds up stochastic. Be that as it may, these two said wonders have electromagnetic root and are not perceptible in the electrostatic approach. In this way, the stage space and the field arrangements of the two methodologies end up not quite the same as each other, expectedly. As a result of the intensification of the wake wave (longitudinal space charge wave), the electrons are quickened to the high vitality level in the electromagnetic approach. An examination of the vitality appropriation elements of the two methodologies in Fig. 3 affirms the above articulation. Also, time advancement of the vitality range in the electrostatic approach in Fig. 3(b) demonstrates that the shape and width of the vitality appropriation work are unaltered after some time. Then again, the quickened electrons discharged because of the limit nonlinear wave breaking, shape the prolonged tail of the appropriation work. This is showed while the mean width of the vitality dissemination work widens over the time within the sight of the electromagnetic impacts as appeared in Fig. 3(a). Truth be told, because of the laser beat adjustment and scrambling impacts, the quickened electrons can achieve the high energies in the electromagnetic administration. In this way, the length of the vitality circulation tail is longer in this approach. Subsequently, without the electromagnetic impacts, the electrons in the plasma are not warmed; and the nonlinear wave breaking impacts simply prompt the electrons speeding up. On the opposite side, in the electromagnetic administration the electrons are quickened and warmed because of the wave breaking component and the beat electromagnetic development.

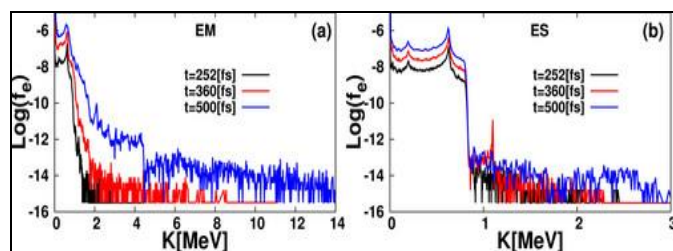


Fig 3: The electron energy distribution function versus electron energy at different times $t = 252$ fs, $t = 360$ fs, and $t = 500$ fs under electromagnetic and (b) electrostatic conditions.

3. Parametric Analyses

In light of what was said in the past segment, it is clear that the limit nonlinear wave breaking can be very delicate to the underlying laser beat parameters. Thus, with a specific end goal to comprehend the impacts of the beat parameters, for example, the beat rise time, beat length, and heartbeat shape on the limit nonlinear wave breaking, distinctive reproductions in light of the PIC recreation are completed without the beat electromagnetic advancements in this segment.

Without the beat electromagnetic advancements, the ponderomotive power assumes a fundamental part in the laser-plasma connection. The ponderomotive power of the laser beat relies upon the beat rise time, as indicated by Eq. (1). The effects of the beat rise time on the limit nonlinear wave breaking are examined by correlation of the association of the three heartbeats with a similar level best length and three distinctive ascent times, for a similar plasma condition in Fig. 4. The adiabatic arrangements and direct PIC reenactment aftereffects of the longitudinal electric fields of the wake wave [panels (a)-(c)] and the stage space of the electrons [panels (d)-(f)] at 100fs in the wake of passing the beats from vacuum-plasma limit are plotted in Fig. 4 for three laser beat with the same 200 fs level best length and the diverse ascent times 30 fs, 50 fs, and 70 fs, individually. It is clear that the adiabatic arrangement and the PIC reproduction consequences of the wake wave are distinctive because of the limit wave breaking. This distinction is considerable for the situation with shorter ascent time and is powerless on the beat with longer ascent time. Rather than the beat with the more extended ascent time, the distinction proceeds over than one wake wavelength for the situation with the shorter ascent time. The stage space plot of the electrons in Fig. 4 exhibits that as the ascent time of the beat diminishes, the limit wave breaking impacts are reinforce. In this manner, more electrons are engaged with the wave breaking, so the confused movements of electrons are predominant and apparent in the stage space design behind the laser beat. Moreover, plainly by diminishing the laser beat rise time, the caught electrons are quickened to the high vitality and the way length of the quickened electrons is expanded. Another marvel which is obvious in the three field examples of the beats with 30 fs, 50 fs, and 70 fs rise times, is variety of the wake field abundance with the expanding of the beat rise-time. For instance on account of the association of the beat with 30fs ascent time, the plentifulness of the wake field diminishes behind the beat; while, the wake field adequacy increments behind the beat wave parcel with 50fs ascent time. This wonder is begun from the coherency of the electrons movement concerning the wake wave toward the finish of the beat. At the point when the electrons (electron pack) waver in thunderous stage regarding the wake field toward the finish of heartbeat, the wake plentifulness is expanded in this area and the other way around.

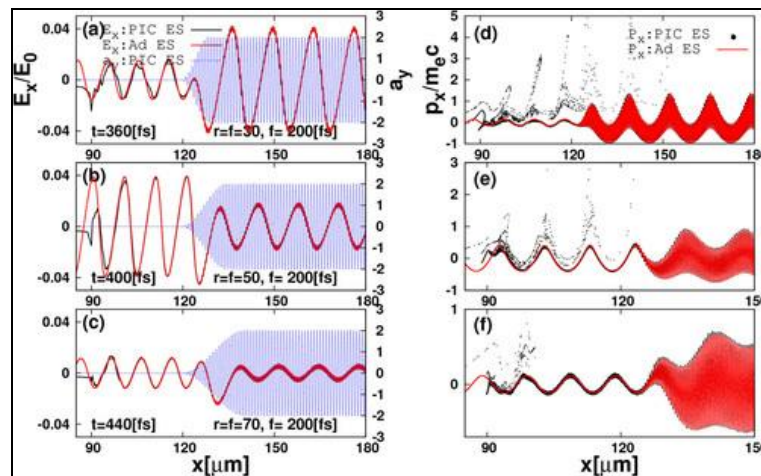


Fig 4: The adiabatic solutions and direct PIC simulation results of the longitudinal electric fields [panels (a)-(c)] and the phase space of the electrons [panels (d)-(f)] for three laser pulse with the same 200fs flat top duration and the different rise times 30fs, 50fs, and 70fs, at 100fs after passing the pulses from vacuum-plasma boundary.

4. Conclusion

This paper centers around the starting point of the nonlinear wave breaking because of the inhomogeneity of the plasma thickness. What's more, impact of the beat parameters, for example, beat rise time, beat level best term and heartbeat length on nonlinear wave softening up vacuum-plasma interface is contemplated by some recreation tests. The nonlinear wave breaking on the vacuum-plasma limit is a non-adiabatic process. Along these lines, the PIC reproduction tests are performed to distinguish the inception of this instrument. In such manner, the time development of the beat field, electrostatic longitudinal electric field and stage space example of the electrons in association of the average limited laser beat with under thick plasma in electromagnetic and electrostatic administrations are contemplated. This examination demonstrates that in the electromagnetic administration adiabatic arrangements (semi static estimation) and PIC comes about are unique in relation to each other toward the finish of heartbeat body (because of the liquid wave breaking impacts), and in the back of the beat on the vacuum-plasma limit (because of the nonlinear wave breaking impacts). On the opposite side in electrostatic administration, clamorous example and contrast between the adiabatic arrangements (semi static estimate) and PIC comes about are seemed just in the back of the beat on the vacuum-plasma interface locally. This demonstrates, the nonlinear wave breaking on the vacuum-plasma interface has electrostatic starting point. Additionally, rather than the liquid wave breaking, which is because of the electromagnetic impacts, just a little bit of the wake wave and a little piece of the electrons are associated with the nonlinear wave breaking system. What's more, the nonlinear wave breaking is privately happened on the vacuum-plasma limit at the essential time of the beat plasma collaboration. Furthermore, the time advancement of the vitality range in these two cases (electrostatic and electromagnetic administrations) are same at the early circumstances of communication, yet those will be diverse as the beat proliferates through the plasma. At the point when impact of the electromagnetic impacts are killed, the nonlinear wave breaking impacts simply prompt speeding

up of the electrons and they are not warmed. Conversely, in the electromagnetic administration, the wave breaking instrument and the beat development cause increasing speed and warming of the electrons.

From that point onward, perceiving of the electrostatic birthplace of the nonlinear wave breaking, the impacts of the beat parameters on the plasma limit nonlinear wave softening are explored up electrostatic administration. The investigation of the beat rise-time variety impacts demonstrate that diminishing of the beat rise time causes augmentation of the limit wave breaking quality. The quantity of the electrons which are associated with the nonlinear wave breaking, most extreme vitality of the quickened electrons and the way length of the quickened electrons in the stage space are expanded by the beat rise-time diminishing. Along these lines, the riotous movements of electrons are prevailing and clear in the stage space design behind the laser beat for the situation with the shorter ascent time. Another marvel obvious in the field designs is that, because of the coherency of the electrons movement as for the wake wave toward the finish of heartbeat, the wake field abundance is fluctuated with the expanding of the beat rise-time. At the point when the electrons (electron bundle) sway in full stage regarding the wake field toward the finish of heartbeat, the wake adequacy is expanded in this area and the other way around.

The impact of the variety of the beat level best span on the nonlinear wave breaking is contemplated by the adiabatic examination and the PIC recreation of collaboration of the three laser beats, with a similar ascent times and the three distinctive level best term times, with a similar plasma thickness profile. The stage space and field designs demonstrate that diminishment of the beat level best span time (the beat length) causes addition of the wake field adequacy behind the beat and diminishing of the quickened electron's way length. Along these lines, the littler piece of the electrons and the littler segment of the wake wave are engaged with the nonlinear wave breaking and impacts of this wave breaking are limited by diminishing of the beat length.

At last, reproduction comes about and adiabatic arrangements of cooperation of the two heartbeats, with various ascent time

and without level best part, with a lofty plasma limit are dissected in electrostatic administration. It is showed up from comes about, the little piece of the electrons and the little segment of the wake wave are engaged with the nonlinear wave softening up the case with the extensive ascent time. Along these lines, the nonlinear wave breaking happens at the vacuum-plasma interface marginally, and PIC recreation comes about and adiabatic arrangement are around coordinated for the entire collaboration time for this situation. As opposed to the previous case, for the situation with the short ascent time, the nonlinear wave breaking impacts are showed up finished the one wake wavelength after the beat proliferation through the plasma limit. Our work can likewise be fascinating to think about the laser plasma situations including the creation of high-vitality electrons and protons [31-34].

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