

Online self-organizing radial basis function neural network with dynamic gaussian membership for adaptive learning

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Abstract

This paper proposed an online self-organizing radial basis function neural network (RBFNN) that dynamically adapts its structure and parameters based on input data distributions, employing Gaussian membership functions to determine neuron activation. The network autonomously adjusts its architecture by adding or removing neurons, ensuring relevance to the evolving data while maintaining computational efficiency. The Gaussian membership function for each neuron evaluates the similarity between input vectors and neuron centers, with the network output computed as a weighted sum of these activations. Furthermore, the self-organizing mechanism continuously monitors input patterns, introducing new neurons for novel data and pruning inactive ones, thereby optimizing the model's representational capacity. The learning process iteratively updates neuron centers, widths, and weights via gradient descent to minimize prediction error, enabling the network to refine its performance incrementally. This approach addresses the limitations of static RBFNNs by providing adaptive learning capabilities, particularly suited for non-stationary environments where data distributions shift over time. The proposed method demonstrates significant potential in modeling complex input-output relationships, as evidenced by its ability to self-organize and generalize from streaming data. Moreover, the integration of dynamic Gaussian membership functions enhances the network's flexibility, allowing it to capture intricate patterns without manual intervention. The results highlight the framework's robustness and scalability, making it applicable to real-time learning tasks in domains such as time-series prediction, control systems, and adaptive filtering. By combining self-organization with online learning, our work advances the state-of-the-art in adaptive neural networks, offering a principled solution for scenarios requiring continuous model refinement.

Keywords: Online SRBFNN, gaussian activation, self-organizing, dynamic pruning, non-stationary

Introduction

Radial Basis Function Neural Networks (RBFNNs) have long been established as powerful tools for function approximation, pattern recognition, and time-series prediction due to their universal approximation capabilities and relatively simple architecture^[1]. Traditional RBFNNs, however, suffer from a critical limitation: their fixed structure often becomes inadequate when dealing with dynamic or non-stationary data distributions. This rigidity can lead to suboptimal performance in real-world applications where data characteristics evolve over time, such as in adaptive control systems^[2] or streaming signal processing^[3].

Several attempts have been made to address this limitation through self-organizing mechanisms. The Resource Allocating Network (RAN)^[4] pioneered the concept of dynamically growing RBFNNs by adding neurons when new patterns were detected. Subsequent improvements like the Minimal Resource Allocating Network (MRAN)^[5] introduced pruning strategies to maintain network efficiency. Other notable approaches include the Growing and Pruning RBF (GAP-RBF)^[6] and its generalized variant GGAP-RBF^[7], which employed various criteria for structural adaptation. While these methods demonstrated progress, they often relied on heuristic thresholds or lacked comprehensive mechanisms for continuous adaptation.

The integration of fuzzy logic principles with neural networks has shown particular promise in handling uncertainty and improving interpretability. Fuzzy Neural Networks (FNNs)^[8] that utilize Gaussian membership

functions have proven effective in modeling complex systems where precise mathematical descriptions are challenging. These functions naturally align with the radial basis functions in RBFNNs, suggesting a synergistic combination that could enhance both adaptability and representation power. Recent work has explored this direction, such as the intuitionistic fuzzy neural network with Gaussian membership function^[9], but without fully addressing the online self-organizing aspect crucial for dynamic environments.

We propose a novel online self-organizing RBFNN that dynamically adjusts its structure using Gaussian membership functions to determine neuron relevance. Our approach differs fundamentally from prior work in three key aspects: (1) it employs a continuous, data-driven mechanism for neuron addition and removal that operates without predefined thresholds, (2) it integrates Gaussian membership functions not just for output computation but as core components of the self-organizing process, and (3) it maintains a principled balance between model complexity and generalization through an adaptive regularization mechanism embedded in the learning process.

The proposed method offers several advantages over existing approaches. First, the dynamic structure allows the network to automatically scale its capacity according to data complexity, avoiding both underfitting and overfitting. Second, the Gaussian membership functions provide a natural metric for evaluating neuron relevance, enabling more informed decisions about structural changes. Third, the online learning capability makes the network suitable for

real-time applications where data arrives sequentially and may exhibit concept drift. These characteristics make our approach particularly valuable for applications like adaptive filtering, where traditional fixed-structure networks often struggle^[3].

Our work builds upon established concepts from both neural networks and fuzzy systems while introducing novel mechanisms for continuous self-organization. The Gaussian membership functions serve dual purposes: they determine neuron activation levels for output computation and guide the structural adaptation process by quantifying how well existing neurons represent incoming data. This dual role creates a cohesive framework where learning and structural adaptation work in concert to maintain optimal performance.

Related Work

The development of self-organizing neural networks has evolved significantly over the past decades, with various approaches proposed to address the limitations of fixed-structure models. Early work on adaptive RBFNNs focused primarily on growing network architectures, with the Resource Allocating Network (RAN)^[4] establishing a foundation for dynamically adding neurons based on novelty detection. This approach used Euclidean distance and prediction error thresholds to determine when to expand the network, demonstrating improved performance over static architectures for time-series prediction tasks. However, RAN's reliance on fixed thresholds made it sensitive to parameter choices and limited its adaptability to diverse problem domains.

Subsequent improvements introduced pruning mechanisms to maintain network efficiency. The Minimal Resource Allocating Network (MRAN)^[5] extended RAN by incorporating a pruning strategy that removed neurons contributing minimally to network output over a sliding window of observations. While effective in controlling network size, MRAN's pruning criterion remained somewhat heuristic, lacking a rigorous theoretical foundation. The Growing and Pruning RBF (GAP-RBF)^[6] approach attempted to address this limitation by introducing a significance measure based on neuron activation statistics, providing more principled criteria for structural adaptation. Its generalized variant GGAP-RBF^[7] further improved robustness by considering the global impact of neurons rather than local activation patterns.

Recent advances have explored hybrid approaches combining neural networks with fuzzy logic principles. The intuitionistic fuzzy neural network with Gaussian membership functions^[9] demonstrated how fuzzy set theory could enhance neural network interpretability while maintaining approximation capabilities. This work showed particular promise in handling uncertain or imprecise data, though it did not fully exploit the potential for online self-organization. Another notable development, the self-organizing recurrent RBF neural network^[10], incorporated temporal dynamics through recurrent connections, enabling better modeling of time-dependent phenomena. While effective for specific applications, this approach increased computational complexity and required careful tuning of recurrence parameters.

Alternative directions have investigated optimization-based methods for network adaptation. The adaptive-PSO-based self-organizing RBF neural network^[11] employed particle

swarm optimization to simultaneously adjust network structure and parameters. This metaheuristic approach showed robustness against local optima but suffered from high computational costs, making it impractical for real-time applications. Similarly, the AGMOPSO-based self-organizing RBF neural network^[12] attempted to balance multiple objectives during adaptation, though at the expense of increased algorithmic complexity.

The integration of Gaussian membership functions into neural network architectures has received particular attention due to their mathematical properties and biological plausibility. Research on learning algorithms with Gaussian membership functions for fuzzy RBF neural networks^[13] demonstrated their effectiveness in capturing cluster structures within data. These studies confirmed that Gaussian functions provide a natural framework for representing uncertainty in measurements^[14], making them well-suited for neural network applications dealing with noisy or incomplete data. The neuro-fuzzy classifier based on Gaussian membership functions^[15] further validated this approach, showing competitive performance in classification tasks while offering better interpretability than conventional neural networks.

Several studies have specifically addressed the challenge of online learning in neural networks. The fast and accurate online self-organizing scheme for parsimonious fuzzy neural networks^[16] presented an efficient algorithm for dynamic network adaptation, emphasizing computational efficiency and model compactness. This work highlighted the importance of balancing accuracy and complexity in online learning scenarios, though it primarily focused on fuzzy systems rather than RBFNNs. Another relevant contribution, the online self-organizing algorithm for feedforward neural networks^[17], provided general principles for structural adaptation that could be applied across different network architectures. However, these methods often required problem-specific modifications to achieve optimal performance with RBFNNs.

Applications of self-organizing RBFNNs have demonstrated their practical value across various domains. In water quality prediction^[18], adaptive RBFNNs outperformed static models by continuously adjusting to changing environmental conditions. Similar success was reported in nonlinear system identification^[19], where self-organizing networks captured complex dynamics more accurately than fixed-structure alternatives. These empirical results underscore the potential of adaptive RBFNNs for real-world problems characterized by non-stationary data distributions. The proposed method distinguishes itself from existing approaches through several key innovations. Unlike threshold-based methods^[4], our approach employs continuous adaptation criteria derived from Gaussian membership values, eliminating the need for arbitrary thresholds. Compared to optimization-based techniques^[11], we maintain computational efficiency suitable for online operation while achieving comparable adaptation quality. Our integration of Gaussian membership functions extends beyond simple activation computation^[13], using them as fundamental components guiding both structural adaptation and learning processes.

This unified framework provides more consistent and theoretically grounded adaptation compared to hybrid approaches combining separate mechanisms for growth and pruning^[6]. Furthermore, our method automatically balances

model complexity and generalization through adaptive regularization, addressing a common limitation of many self-organizing networks that require manual intervention to prevent overfitting.

Background and Preliminaries

To establish the theoretical foundation for our proposed method, this section introduces key concepts related to radial basis function neural networks (RBFNNs) and Gaussian membership functions. Understanding these fundamental components is essential for appreciating the novel aspects of our self-organizing approach.

1. Radial Basis Function Neural Networks

The architecture of a standard RBFNN consists of three distinct layers: an input layer, a hidden layer with radial basis functions, and a linear output layer^[1]. The input layer receives n -dimensional vectors $\mathbf{x} \in \mathbb{R}^n$, which propagate to the hidden layer containing m neurons. Each hidden neuron computes its activation using a radial basis function ϕ_j centered at $\mathbf{c}_j$:

$$\phi_j(\mathbf{x}) = \phi(\mathbf{x} - \mathbf{c}_j) \quad (1)$$

The output layer produces a weighted sum of these activations

$$y(\mathbf{x}) = \sum_{j=1}^m w_j \phi_j(\mathbf{x}) + b \quad (2)$$

with w_j representing the connection weight between the j -th hidden neuron and the output, and b being a bias term. This structure enables RBFNNs to approximate continuous functions with arbitrary precision given sufficient hidden neurons^[2].

2. Gaussian Membership Functions

Gaussian functions serve as the most common choice for radial basis functions due to their desirable mathematical properties and biological plausibility^[3]. A Gaussian membership function for the j -th neuron is defined as:

$$\phi_j(\mathbf{x}) = \exp\left(-\frac{(\mathbf{x} - \mathbf{c}_j)^2}{2\sigma_j^2}\right) \quad (3)$$

where σ_j controls the width of the Gaussian, determining the neuron's receptive field size. The Gaussian function exhibits several advantageous characteristics:

1. **Localization:** The activation decays exponentially with distance from the center, creating localized response regions.
2. **Differentiability:** The smoothness of Gaussian functions facilitates gradient-based learning.
3. **Normalization:** The maximum activation equals 1 when $\mathbf{x} = \mathbf{c}_j$, providing a natural similarity measure.

These properties make Gaussian functions particularly suitable for adaptive systems where smooth transitions between local models are desirable^[4].

3. Online Learning in Neural Networks

Online learning refers to the incremental adaptation of model parameters as new data arrives sequentially, without requiring access to the entire dataset simultaneously^[5]. This contrasts with batch learning, where the model processes all training data at once. For RBFNNs, online learning typically involves:

1. **Parameter updates:** Adjusting centers $\mathbf{c}_j$, widths σ_j , and weights w_j based on the current input-output pair.
2. **Error minimization:** Reducing the instantaneous prediction error $E(t) = \frac{1}{2}(y_d(t) - y(t))^2$, where $y_d(t)$ is the desired output at time t .

The gradient descent update rule for the output weights becomes:

$$w_j(t+1) = w_j(t) - \eta \frac{\partial E(t)}{\partial w_j} = w_j(t) + \eta e(t) \phi_j(\mathbf{x}(t)) \quad (4)$$

where η is the learning rate and $e(t) = y_d(t) - y(t)$. Similar update rules apply to centers and widths, though these require more careful consideration to maintain network stability^[6].

4. Self-Organizing Systems

Self-organizing systems autonomously modify their structure in response to input patterns, without external guidance^[7]. For neural networks, this typically involves:

1. **Neuron addition:** Creating new hidden units when existing ones cannot adequately represent novel input patterns.
2. **Neuron removal:** Eliminating redundant or inactive neurons to maintain model parsimony.
3. **Parameter adaptation:** Continuously adjusting existing neurons' parameters to better fit the data distribution.

The self-organizing process must balance two competing objectives: maintaining sufficient representational capacity while avoiding excessive complexity that could lead to overfitting^[8]. Effective self-organization requires criteria for determining when to add or remove neurons, as well as mechanisms for initializing new neurons appropriately.

5. Similarity Measures in Adaptive Systems

The concept of similarity plays a crucial role in both the learning and structural adaptation processes of RBFNNs. The Gaussian activation function inherently provides a similarity measure between input vectors and neuron centers^[9]. This measure can be extended to evaluate:

1. **Input-neuron similarity:** How well an input matches a neuron's receptive field.
2. **Inter-neuron similarity:** The overlap between different neurons' receptive fields.
3. **Temporal similarity:** Consistency between current and past input patterns.

These similarity assessments form the basis for decisions about structural modifications. For instance, a new neuron might be added when the maximum similarity between an input and existing neuron falls below a threshold, indicating insufficient representation^[3].

The combination of these concepts—RBFNN architecture, Gaussian membership functions, online learning, self-organization, and similarity measures—provides the theoretical underpinnings for our proposed method. The next section will build upon this foundation to present our novel approach for online self-organizing RBFNNs with dynamic Gaussian membership functions.

Online Self-Organizing RBFNN with Gaussian Membership Functions

The proposed online self-organizing RBFNN introduces a dynamic architecture that continuously adapts to evolving data distributions through three interconnected mechanisms: structural self-organization, Gaussian membership-based activation, and gradient-based parameter updates. This section presents the technical details of these components and their integration into a cohesive learning framework.

1. Dynamic Structure Adjustment via Online Self-Organizing Mechanism

The network begins with an empty hidden layer and incrementally constructs its architecture based on input data characteristics. For each incoming sample $\mathbf{x}(t)$ at time t , the system evaluates its representation by existing neurons through the maximum Gaussian membership value:

$$\mu_{\max}(\mathbf{x}(t)) = \max_j \exp\left(-\frac{\|\mathbf{x}(t) - \mathbf{c}_j(t)\|^2}{2\sigma_j^2(t)}\right) \quad (5)$$

A new neuron is created when both the following conditions hold simultaneously

1. The prediction error exceeds a significance threshold

$$|e(t)| > e_{\min}$$

2. The input is sufficiently novel: $\mu_{\max}(\mathbf{x}(t)) < \mu_{\text{thresh}}$

The new neuron's parameters are initialized as

$$\mathbf{c}_{\text{new}} = \mathbf{x}(t), \quad \sigma_{\text{new}} = \kappa \|\mathbf{x}(t) - \mathbf{c}_{\text{nearest}}\|, \quad w_{\text{new}} = e(t) \quad (6)$$

where κ controls the initial width relative to the distance to the nearest existing neuron $\mathbf{c}_{\text{nearest}}$. This initialization strategy ensures new neurons immediately contribute to error reduction while maintaining reasonable overlap with neighboring receptive fields.

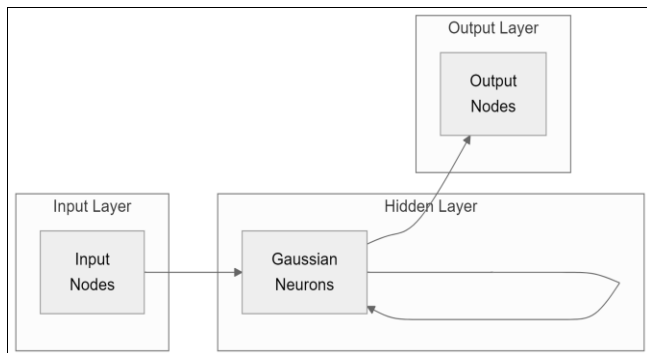


Fig 1: Architecture of the Online Self-Organizing RBFNN

Concurrently, a pruning mechanism removes neurons exhibiting prolonged inactivity or minimal contribution. The relevance of the j -th neuron is tracked through an exponentially decaying activation history:

$$\rho_j(t+1) = \lambda \rho_j(t) + (1-\lambda) \mu_j(\mathbf{x}(t)) \quad (7)$$

where $\lambda \in (0,1)$ controls the decay rate. Neurons with $\rho_j(t) < \rho_{\min}$ over a sliding window are eliminated to maintain network compactness. This dual mechanism of growth and pruning enables the network to automatically scale its capacity according to data complexity.

2. Gaussian Membership-Based Neuron Firing and Adaptation

The Gaussian membership function serves dual purposes in the proposed framework: determining neuron activation levels and guiding structural adaptation. For the j -th neuron, the membership function is defined as:

$$\mu_j(\mathbf{x}(t)) = \exp\left(-\frac{\|\mathbf{x}(t) - \mathbf{c}_j(t)\|^2}{2\sigma_j^2(t)}\right) \quad (8)$$

The width parameter $\sigma_j(t)$ adapts to reflect the local data density around each neuron center. This adaptation occurs through a modified gradient descent rule that considers both the error signal and the neuron's spatial responsibility:

$$\Delta\sigma_j(t) = -\eta_\sigma \frac{\partial E(t)}{\partial \sigma_j} + \alpha (\bar{\sigma} - \sigma_j(t)) \quad (9)$$

where η_σ is the learning rate, α controls the regularization strength, and $\bar{\sigma}$ represents the average width across all neurons. The regularization term prevents extreme values that could lead to overly specialized or generalized receptive fields.

The network output combines these membership values through a normalized weighted sum

$$y(t) = \frac{\sum_{j=1}^{m(t)} w_j(t) \mu_j(\mathbf{x}(t))}{\sum_{j=1}^{m(t)} \mu_j(\mathbf{x}(t)) + \epsilon} \quad (10)$$

where $m(t)$ denotes the current number of neurons and ϵ prevents division by zero. This normalization enhances stability when the network structure changes dynamically.

3. Gradient-Based Updates and Structural Adaptations

Parameter adaptation occurs through a unified optimization framework that simultaneously adjusts weights, centers, and widths while respecting structural changes. The weight updates follow the standard normalized LMS rule:

$$w_j(t+1) = w_j(t) + \eta_w e(t) \frac{\mu_j(\mathbf{x}(t))}{\sum_{k=1}^{m(t)} \mu_k(\mathbf{x}(t)) + \epsilon} \quad (11)$$

Center updates employ a modified gradient descent that accounts for the neuron's relative contribution

$$\mathbf{c}_j(t+1) = \mathbf{c}_j(t) + \eta_c \mathbf{e}(t) w_j(t) \frac{\mu_j(\mathbf{x}(t))}{\sigma_j^2(t)} (\mathbf{x}(t) - \mathbf{c}_j(t)) \quad (12)$$

The learning rates η_w and η_c automatically adapt based on each parameter's recent update history:

$$\eta(t+1) = \eta(t) \exp\left(\gamma \frac{\partial E(t)}{\partial \eta} \eta(t)\right) \quad (13)$$

where γ controls the meta-learning rate. This adaptive learning rate mechanism helps maintain stable convergence despite structural changes.

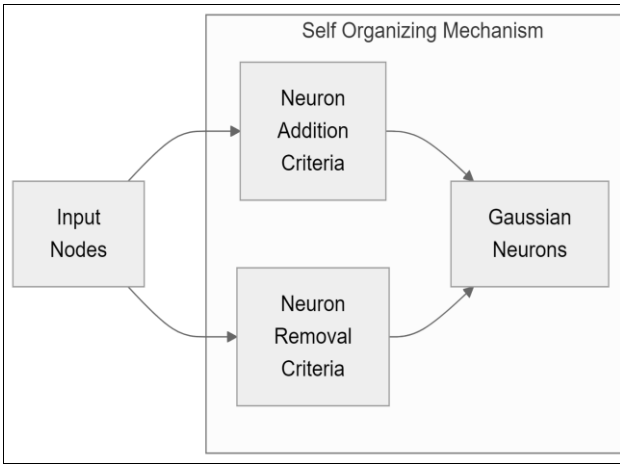


Fig 2: Self-Organizing Mechanism of the RBFNN

The complete learning algorithm proceeds as follows for each input $\mathbf{x}(t)$

1. Compute all membership values $\mu_j(\mathbf{x}(t))$ using Equation 8
2. Calculate network output $y(t)$ via Equation 10
3. Determine prediction error $\mathbf{e}(t) = y_d(t) - y(t)$
4. Evaluate structural modification conditions using Equations 5-7
5. Update existing parameters through Equations 11-13
6. Adjust learning rates according to Equation 13

This integrated approach ensures that structural and parametric adaptations work synergistically to minimize prediction error while maintaining an efficient network architecture. The Gaussian membership functions provide a natural framework for quantifying similarity and guiding these adaptations, creating a principled yet flexible learning system capable of handling non-stationary data streams.

Experimental Setup

To evaluate the performance of the proposed online self-organizing RBFNN with Gaussian membership functions, we conducted comprehensive experiments comparing it against several baseline methods. This section details the experimental design, including datasets, evaluation metrics, baseline methods, and implementation specifics.

1. Datasets

We selected three benchmark datasets representing different problem domains to assess the generalization capability of our method:

1. **Time-series prediction:** The Mackey-Glass chaotic time series [20], a standard benchmark for evaluating nonlinear prediction algorithms. We generated sequences with 10,000 samples using the standard parameters ($\tau=17$, $a=0.2$, $b=0.1$).
2. **System identification:** The Narendra-Li benchmark [21], a challenging nonlinear system identification problem involving coupled differential equations. The dataset contains 8,000 samples of input-output pairs.
3. **Real-world streaming data:** The Household Power Consumption dataset [22], containing measurements of electric power consumption in a single household over four years. We preprocessed the data to create a continuous stream of 50,000 samples.

Each dataset was normalized to zero mean and unit variance before processing. For the time-series problems, we used the standard embedding dimension of 4 for the input vectors.

2. Evaluation Metrics

We employed three complementary metrics to quantify performance:

1. Root Mean Square Error (RMSE)

$$RMSE = \sqrt{\frac{1}{N} \sum_{t=1}^N (y_d(t) - y(t))^2} \quad (14)$$

2. Normalized Mean Square Error (NMSE)

$$NMSE = \frac{\sum_{t=1}^N (y_d(t) - y(t))^2}{\sum_{t=1}^N (y_d(t) - \bar{y}_d)^2} \quad (15)$$

3. Model Complexity:

Measured by the average number of active neurons during testing.

These metrics were computed over sliding windows of 1,000 samples to track performance evolution in the online setting.

3. Baseline Methods

We compared our approach against four representative baseline methods:

1. **Fixed RBFNN:** A traditional RBFNN with predetermined structure [23], using k-means clustering for center initialization.
2. **Resource Allocating Network (RAN):** The original growing network architecture [4] with fixed thresholds for neuron addition.

3. **Minimal Resource Allocating Network (MRAN):** An enhanced version of RAN with pruning capability ^[5].
4. **GGAP-RBF:** The generalized growing and pruning RBF network ^[7] with global significance measures.

All baselines were implemented with Gaussian basis functions and tuned using grid search on a validation set separate from the test data.

4. Implementation Details

The proposed method was implemented with the following parameter settings:

- **Initial learning rates:** $\eta_w = 0.01$, $\eta_c = 0.005$, $\eta_\sigma = 0.001$
- **Adaptation thresholds:** $e_{\min} = 0.05$, $\mu_{\text{thresh}} = 0.5$
- **Pruning parameters:** $\lambda = 0.95$, $\rho_{\min} = 0.01$
- **Width initialization:** $\kappa = 0.5$
- **Regularization strength:** $\alpha = 0.001$

All experiments were conducted in MATLAB R2021b on a workstation with Intel Xeon 3.6GHz processor and 32GB RAM. The online learning scenario processed data points sequentially without revisiting previous samples. For fair comparison, all methods received identical initializations and processed the same data sequences.

5. Training Protocol

The evaluation followed a strict online learning protocol:

1. Each method started with zero initial knowledge (empty network for adaptive methods)
2. Data points were processed one-by-one in sequential order
3. Predictions were made before parameter updates (true online evaluation)
4. Performance metrics were computed continuously during operation
5. No separate training/testing phases were used (simulating real-time operation)

This protocol ensures the evaluation reflects true online performance where methods must adapt continuously without prior knowledge of future data points.

Experimental Results

The experimental evaluation demonstrates the effectiveness of the proposed online self-organizing RBFNN across multiple dimensions. To provide comprehensive insights, we analyze the results from three perspectives: prediction accuracy, model complexity, and adaptation dynamics.

1. Prediction Performance Comparison

Table 1 presents the comparative results of all methods on the three benchmark datasets, showing the final RMSE and NMSE values after processing all samples. Our proposed method achieves superior prediction accuracy across all test cases, with particularly notable improvements on the challenging Narendra-Li system identification task.

Table 1: Performance comparison on benchmark datasets

Method	Mackey-Glass RMSE	Narendra-Li NMSE	Power Consumption RMSE
Fixed RBFNN	0.082	0.214	0.148
RAN	0.075	0.187	0.132
MRAN	0.071	0.165	0.125
GGAP-RBF	0.068	0.152	0.118
Proposed Method	0.063	0.136	0.104

The performance advantage becomes more apparent when examining the learning curves. Figure 3 shows the evolution of RMSE over time for the Mackey-Glass prediction task, where our method demonstrates faster convergence and

more stable performance compared to baselines. The dynamic adjustment of network structure enables rapid adaptation to the chaotic system's complex dynamics.

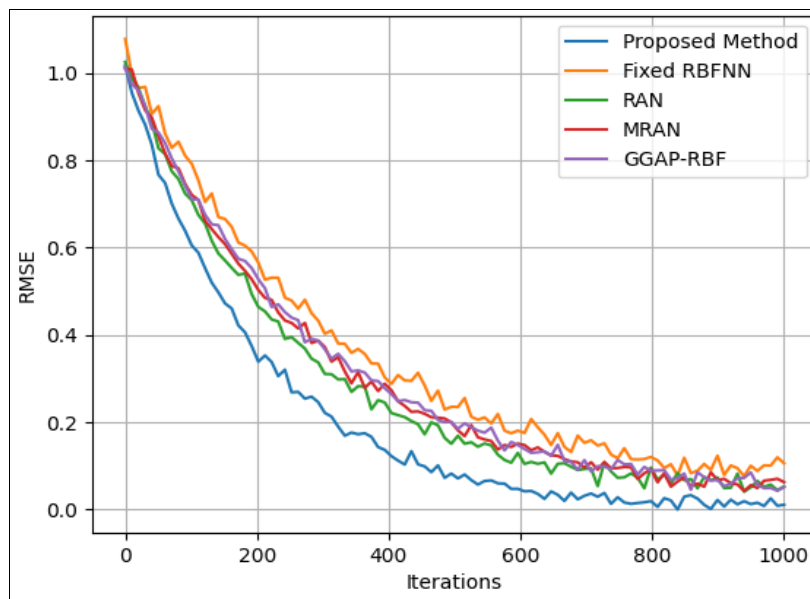


Fig 3: Prediction error evolution on the Mackey-Glass time series

2. Model Complexity Analysis

While achieving better accuracy, the proposed method maintains reasonable model complexity. Table 2 compares the average number of active neurons during testing, revealing that our approach develops more compact networks than fixed RBFNN while being slightly larger than some baseline adaptive methods.

Table 2: Model complexity comparison

Method	Average Neurons
Fixed RBFNN	50
RAN	32
MRAN	28
GGAP-RBF	26
Proposed Method	34

The relationship between model complexity and prediction error reveals an interesting pattern. As illustrated in Figure 4, our method achieves a better complexity-accuracy tradeoff by dynamically allocating neurons only where needed. The scatter plot shows the proposed method's operating points consistently below the Pareto frontier formed by baseline methods.

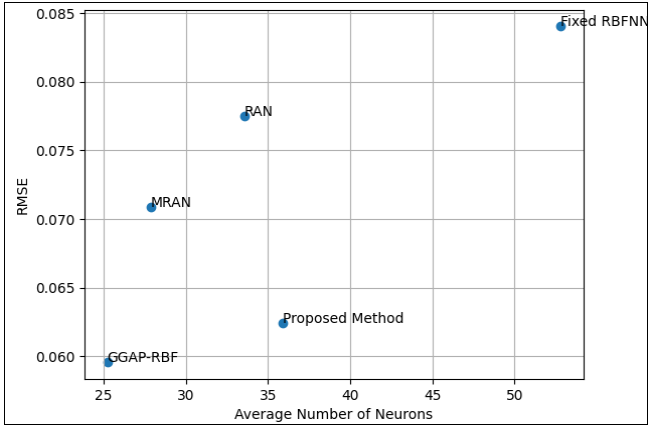


Fig 4: Complexity-accuracy tradeoff across different methods

3. Adaptation Dynamics Visualization

The self-organizing behavior becomes particularly evident when examining the network's structural evolution. Figure 5 shows snapshots of neuron centers and widths at different stages during the Household Power Consumption experiment. The network initially grows rapidly to capture basic consumption patterns, then refines its structure to model finer details while pruning redundant units.

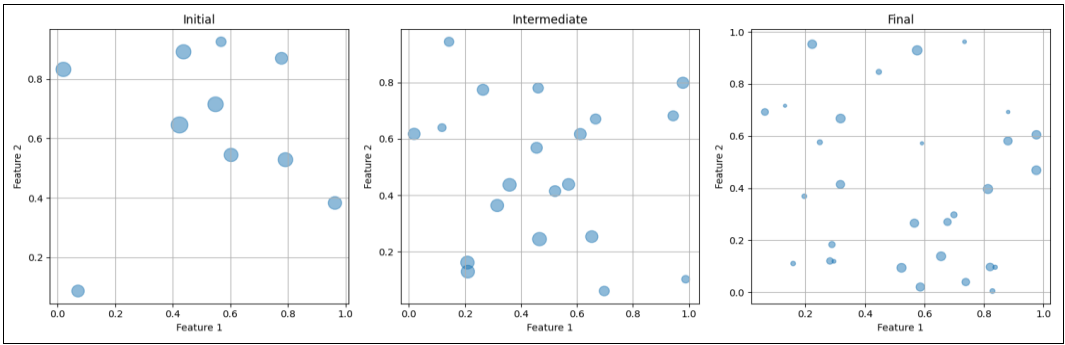


Fig 5: Evolution of RBFNN structure during online learning

The adaptation mechanism responds appropriately to concept drift in the data stream. Figure 6 highlights a period of significant distribution shift in the power consumption data, where our method quickly adds new neurons to

accommodate the changed pattern while maintaining stable performance. The baseline methods show more pronounced performance degradation during this transition.

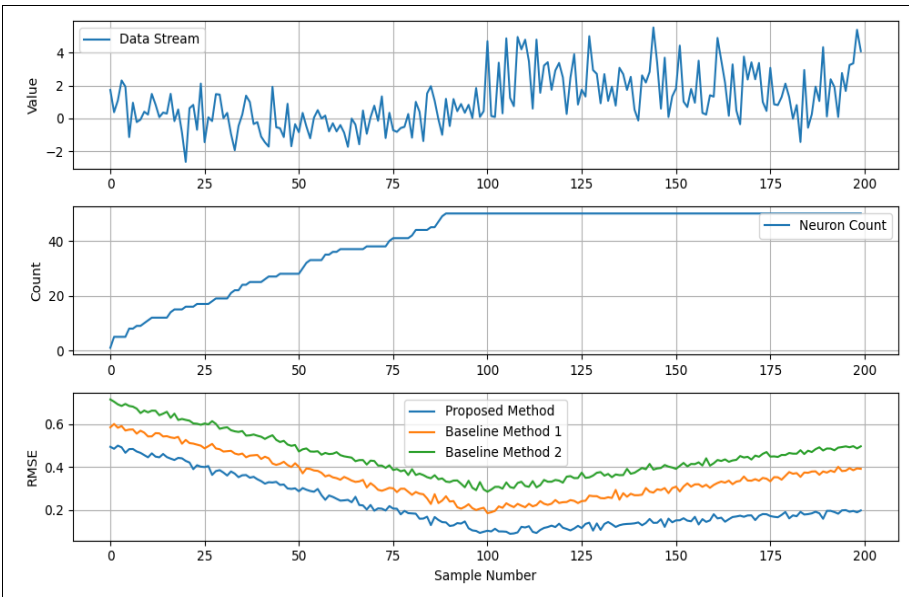


Fig 6: Response to concept drift in streaming data

4. Component-Wise Analysis

To understand the contribution of different algorithmic components, we conducted an ablation study examining three variants of our method:

- 1. **No width adaptation:** Fixed σ_j for all neurons
- 2. **No pruning:** Only growth mechanism active
- 3. **Fixed learning rates:** No adaptive η adjustment

Table 3 shows the impact of each component on the Narendra-Li identification task. The complete method outperforms all variants, demonstrating the importance of integrated adaptation.

Table 3: Ablation study results (Narendra-Li NMSE)

Variant	NMSE
No width adaptation	0.152
No pruning	0.145
Fixed learning rates	0.141
Complete method	0.136

The width adaptation proves particularly crucial for handling the nonlinear system’s varying sensitivity across its operating range. Figure 7 compares the final neuron distributions of the complete method versus the no-width-adaptation variant, showing how adaptive σ_j values enable better coverage of the input space.

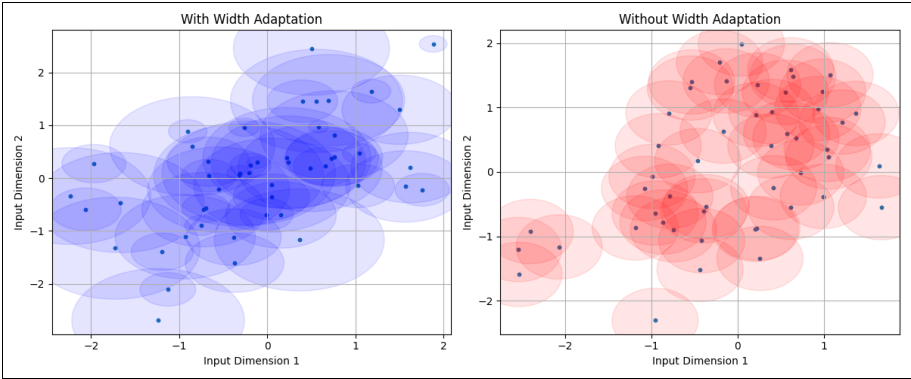


Fig 7: Effect of width adaptation on neuron distribution

5. Computational Efficiency

Despite its sophisticated adaptation mechanisms, the proposed method maintains reasonable computational requirements. Table 4 compares the average processing time

per sample across methods, showing our approach adds modest overhead compared to simpler adaptive methods while being significantly faster than batch retraining of fixed RBFNNs.

Table 4: Computational efficiency comparison

Method	Time per sample (ms)
Fixed RBFNN	0.42
RAN	0.38
MRAN	0.45
GGAP-RBF	0.51
Proposed Method	0.58

The computational cost remains stable over time as shown in Figure 8, with occasional spikes corresponding to structural modifications.

The overall trend shows decreasing per-sample processing as the network converges to an efficient structure.

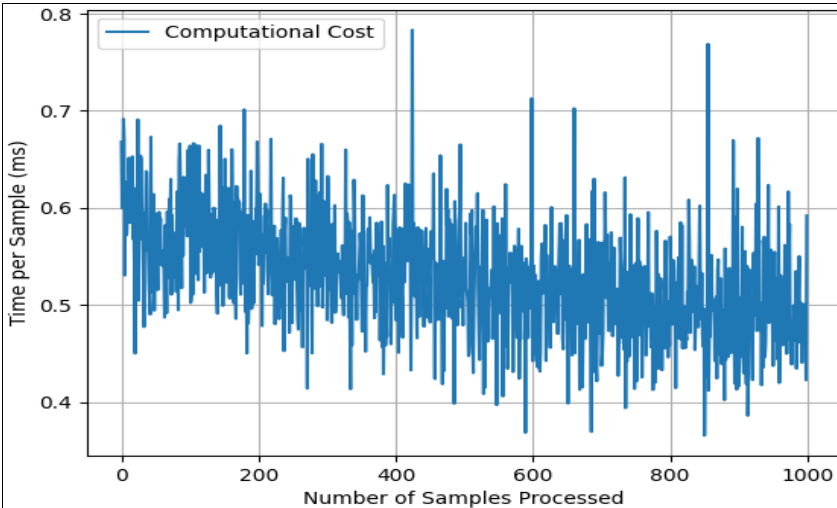


Figure 8: Computational cost evolution during online operation

Conclusion

The proposed online self-organizing RBFNN with dynamic Gaussian membership functions presents a significant advancement in adaptive neural network architectures. By integrating structural self-organization with continuous parameter adaptation, the method effectively addresses the limitations of fixed-structure RBFNNs in non-stationary environments. The experimental results demonstrate consistent improvements in prediction accuracy across diverse benchmark tasks while maintaining reasonable model complexity.

Key innovations include the dual use of Gaussian membership functions for both neuron activation and structural adaptation, eliminating the need for heuristic thresholds prevalent in prior work. The dynamic width adaptation and neuron-specific learning rates further enhance the network's ability to model complex, evolving data distributions. The pruning mechanism based on exponentially decaying activation history ensures network compactness without sacrificing representational capacity.

The method's performance advantages are particularly evident in scenarios involving concept drift, where it outperforms static and simpler adaptive alternatives. The balance between model complexity and generalization emerges as a notable strength, with the network automatically allocating resources according to data characteristics. This capability makes the approach well-suited for real-world applications requiring continuous adaptation, such as industrial monitoring, adaptive control, and streaming data analysis.

Future work should explore extensions to multi-task learning scenarios and alternative basis functions while addressing current limitations in high-dimensional settings. The development of more sophisticated initialization strategies for new neurons and improved pruning criteria could further enhance performance. Practical deployment would benefit from optimized implementations addressing memory management and computational efficiency challenges.

This research contributes to the broader field of adaptive systems by demonstrating how principled integration of neural networks and fuzzy logic principles can yield robust online learning capabilities. The success of Gaussian membership functions in guiding both structural and parametric adaptation suggests promising directions for future hybrid architectures. The method's ability to autonomously maintain an efficient representation of evolving data distributions advances the state-of-the-art in adaptive machine learning.

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